VORTICAL STRUCTURE DEVELOPMENT IN STABLY STRATIFIED TURBULENCE

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Abstract Relation between the power-law transition in the energy spectra and the development of vortical structures for stably stratified turbulence is investigated using direct numerical simulations (DNS) at a resolution of up to 2048^3. The calculation is done by solving the 3D Navier-Stokes equations under the Boussinesq approximation pseudo-spectrally. Using toroidal-poloidal decomposition (Craya-Herring decomposition), the velocity field is divided into the vortex mode (ϕ_1) and the wave mode(ϕ_2). Both the wave and vortex spectra are consistent with a Kolmogorov–like $k^{-5/3}$ range at sufficiently large k. At large scales, and for sufficiently strong stratification, the wave spectra is a steeper $k_\perp^{-2}$, while that for the vortex component is consistent with $k_\perp^{-3}$. Here $k_\perp$ is the horizontally gathered wave numbers. While the Kolmogorov– like spectra are developing, some characteristic vortical structures appear successively.

ENERGY SPECTRA

In the atmosphere and oceans, flows are often stably stratified, and clarifying the mechanism of stratification is a vital problem in the whole geophysical and astrophysical fluid dynamics. In this paper, the energy spectra for forced stably stratified turbulence are investigated numerically using the Direct Numerical Simulations (DNS) with up to 2048^3 grid points ($R_\lambda \sim 300$). The simulation is done by solving the 3D momentum equation under the Boussinesq approximation pseudo-spectrally with stochastic forcing applied to the large horizontal velocity scales

Using toroidal-poloidal decomposition (Craya-Herring decomposition), the velocity field is divided into the vortex mode (ϕ_1) and the wave mode (ϕ_2). With the initial kinetic energy being zero, the ϕ_1 spectra as a function of horizontal wave numbers, $k_\perp = \sqrt{k_x^2 + k_y^2}$, first develops a $k_\perp^{-3}$ spectra for the whole $k_\perp$ range, and then $k_\perp^{-5/3}$ part appears at large $k_\perp$ with rather a sharp transition wave number. **Figure. 1** shows ϕ_1 spectra for $N^2 = 1, 10, 50, 100$ (from the top to the bottom) where N is the Brunt–Väisälä frequency. We can observe that the small $k_\perp$ parts collapse to a single spectrum of $\sim k_\perp^{-3}$, while the large $k_\perp$ parts have the same slope of $k_\perp^{-5/3}$ but with different coefficients depending on N. For scaling these spectra, we use the following two points as criterions; (1) the large scales do not depend on N, (2) we expect that the Kolmogorov constant is universal for the isotropic subset of anisotropic data, and we propose the functional form as;

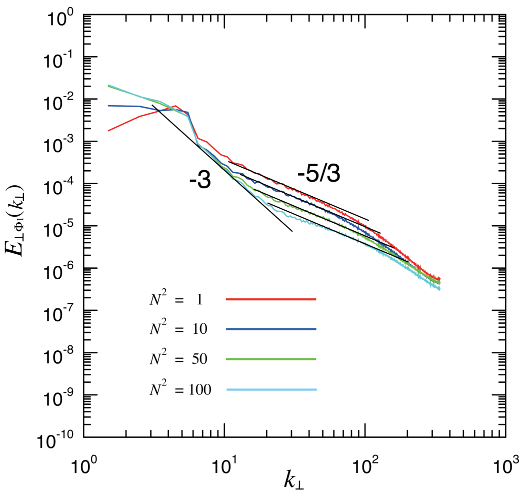

Figure 1. $\phi_1$spectra as a function of $k_\perp$ for $N^2 = 1, 10, 50, 100$. [4]

$$E_{\perp\Phi_1}(k_\perp) = \begin{cases} \alpha \eta_{\perp\Phi_1}^{2/3} k_\perp^{-3} & (k_\perp < k_c) \\ C_K \varepsilon_{\perp\Phi_1}^{2/3} k_\perp^{-5/3} & (k_\perp > k_c) \end{cases} , \tag{1}$$

where

$$\varepsilon_{\perp\Phi_i} = 2\nu \int_0^\infty k_\perp^2 E_{\perp\Phi_i}(k_\perp) \mathrm{d}k_\perp , \quad \eta_{\perp\Phi_i} = 2\nu \int_0^\infty k_\perp^4 E_{\perp\Phi_i}(k_\perp) \mathrm{d}k_\perp \quad (i = 1, 2). \tag{2}$$

The transition of energy spectrum from $k_\perp^{-3}$ to $k_\perp^{-5/3}$ has been observed in the atmosphere[1] and in the ocean [2],[3]. Such a transition is possible when the Ozmidov buoyancy scale $L_O = (\varepsilon/N^3)^{1/2}$ is larger than the dissipation scale $L_K = (\nu^3/\varepsilon)^{1/4}$, with ε the energy dissipation rate, N the Brunt–Väisälä frequency, and ν kinematic viscosity.

VORTICAL STRUCTURES

In the course of development for a stationary shape, the horizontal spectrum undergoes some different stages. At the first stage, it shows a single steep power-law ($k_\perp^{4-5}$). By this time, we observe that many wedge vortices are produced and they move horizontally (like dipoles) in random directions. This stage lasts a long period of time, and then the tail part of the spectrum begins to rise to show the Kolmogorov-type slope ($k_\perp^{-5/3}$). During the time of this stage, the wings of the wedges become thinner and thinner while translating, and finally detach to be almost independent vortex layers. This thinning mechanism makes the vertical shear stronger and eventually local Richardson number small to develop Kelvin-Helmholtz billows. We will show that the horizontal breaking of the Kelvin-Helmholtz billows results in the Kolmogorov-type slope in the spectrum. (**Figure. 2**)

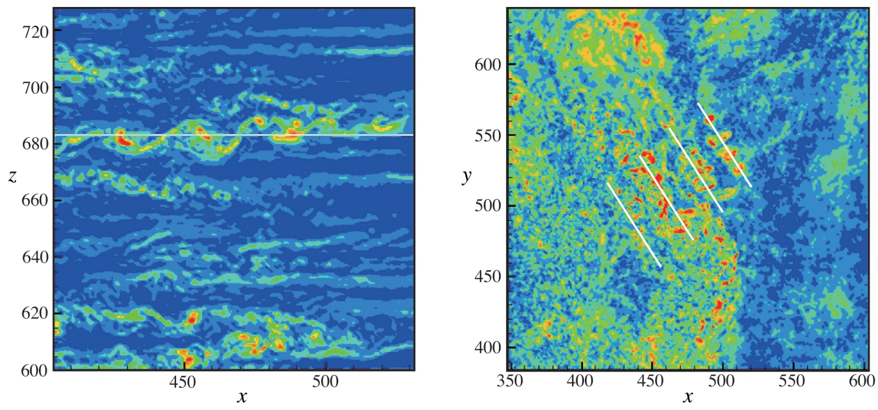

Figure 2. Left: Kelvin-Helmholtz billows in stably stratified turbulence. Right: The horizontal slice near the Kelvin-Helmholtz billow (along the white line in the left figure). The numbers at axes are the grid numbers. [4]

References

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