FROM PERIODIC TO CHAOTIC SELF-SUSTAINING PROCESS IN BOUNDARY-LAYER FLOWS

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Abstract The dynamics on the laminar-turbulent separatrix for boundary-layer flows is investigated numerically in the subcritical regime. In order to prevent the growth of the boundary-layer thickness, constant homogeneous suction is applied at the flat plate leading to a parallel Asymptotic Suction Boundary Layer (ASBL). In the spanwise extended numerical domain the coherent structures found by edge tracking are invariably localised and their dynamics shows bursts followed by spanwise shifts. As the length of the domain is varied, the asymptotic dynamics on the edge switches from being periodic in time to chaotic. In all cases a clear mechanism for the regeneration of streaks and streamwise vortices is identified. These findings are important for a deeper understanding of both subcritical (bypass) transition and the yet not fully clear regeneration cycle in wall turbulence. The flow at hand can serve as a laboratory to identify the various phases in near-wall cycle which can be studied in a clean and isolated way. Furthermore, the identified localised dynamics showing exact periodicity is very interesting from a dynamical-systems point of view.

INTRODUCTION

Near-wall coherent structures such as streaks and quasi-streamwise vortices are an ubiquitous feature of transitional and turbulent wall-bounded shear flows. Their regeneration process is intimately connected with the occurrence of *bursting* events, *i.e.* strong intermittent ejections of low-speed fluid from the wall [5]. We focus on coherent structures as well as bursting events in the framework of subcritical transition. A recent idea specific to subcritical instabilities is to analyse the laminar-turbulent separatrix, the invariant phase-space region separating trajectories that relaminarise from those experiencing turbulent dynamics. Relative attractors on this separatrix are called edge states [6]. They correspond to an intrinsic (metastable) equilibrium regime and are thus crucial for i) understanding the structure of the phase space and ii) identifying the physical mechanisms by which the flow can sustain non-trivial dynamics.

The concept of edge states was recently applied to the Blasius boundary-layer flow over a flat plate [1], where an additional complication is the spatial development of the boundary layer. Parallel flows are much better suited to asymptotical edge tracking since periodic boundary conditions both allow for streamwise periodic structures, and a constant layer thickness (*i.e.* Reynolds number) in the domain. One of the possible ways to achieve a parallel boundary-layer flow is by applying suction at the lower wall to compensate for the spatial growth of the laminar profile. In the case when the suction is constant and homogeneous the boundary-layer thickness eventually saturates and the associated flow is termed Asymptotic Suction Boundary Layer (ASBL).

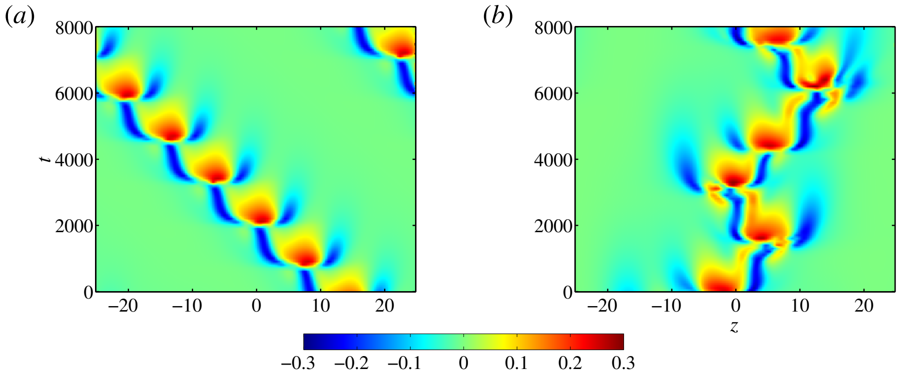

Figure 1. Space-time diagrams for streamwise velocity fluctuations u' averaged in streamwise direction at a fixed wall-parallel plane, for Reynolds number $Re = U_\infty \delta^*/\nu = U_\infty/V_S = 500$. The box size is $(L_x, L_y, L_z) = (6\pi\ \delta^*, 15\ \delta^*, 50\ \delta^*)$. (*a*) Left-going periodic state (L). (*b*) Chaotic edge state in a slightly shorter domain.

SPANWISE LOCALISED EDGE STATES

Edge states in a minimal flow unit ASBL have been discussed in [4]. Here we focus on spanwise extended numerical domains. In this case we obtain three different states (two of them are actually equivalent under a reflection symmetry), which are localised in the spanwise direction and periodic in time. In all three cases the states are dominated by a pair of high- and low-speed streaks which undergo a burst before being translated in the spanwise direction. Depending on the direction of the shift we distinguish between the two symmetry-related states that repeatedly shift towards the right (R) or towards the left (L), and the state that alternates regularly between shifting left and right (LR). This can be clearly seen for the L state in the space-time representation in figure 1(*a*).

The qualitative dynamics between two consecutive bursts appears to be similar in all three cases. Representative snapshots of the cycle are shown in figure 2 for the L state. In the calm phase the state consists of one high-speed streak with two low-speed streaks on each side. One of the low-speed streaks is moderately bent and is flanked by counter-rotating quasi-streamwise vortices, which sustain the streak (lift-up effect). Conversely, the second streak is less bent and is slowly decaying. As the vortices grow in strength they wrap around the streak, tilting in the streamwise direction and causing the streak to bend even further, ultimately leading to its breakdown and creation of a high-speed streak at the same location. In the streak breakdown process the initial streamwise vortices are destroyed and new vortices are recreated in the area around the newly created high-speed streak. The vortices on both sides of the high-speed streak create two low-speed streaks and the loop is closed. The low-speed streak on the left is the active one that will develop instabilities and break down during the next burst, while the right one will slowly decay, resulting in a leftwards shift of the whole structure. This self-sustaining process strongly resembles the regeneration cycle of the near-wall turbulent structures [2].

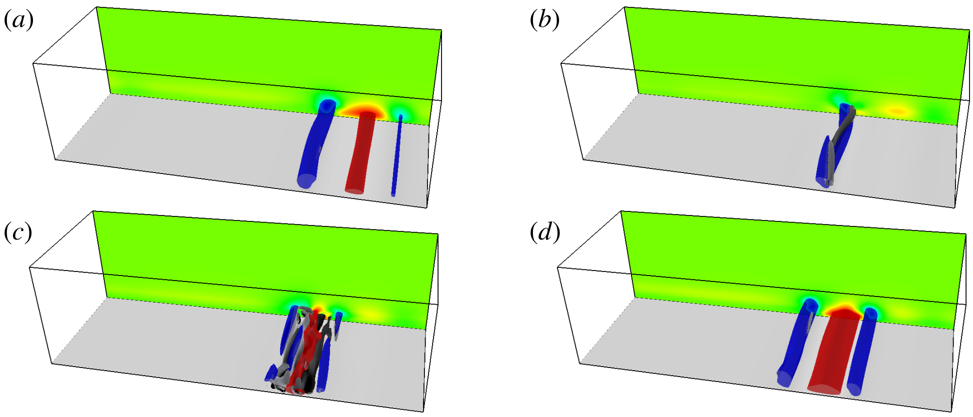

Figure 2. Three-dimensional visualisation during one period of the L state for $Re = 500$. Isosurfaces of streamwise velocity fluctuations $u' = \pm 0.15\ U_\infty$ are coloured in blue (low-speed streak) and red (high-speed streak), respectively. Vortices are visualised using the λ_2 criterion [3] with the isosurface $\lambda_2 = -0.001\ U_\infty^2/\delta^{*2}$ coloured by the streamwise vorticity (grey-scale). The whole computational domain of size $(L_x, L_y, L_z) = (6\pi\ \delta^*, 15\ \delta^*, 50\ \delta^*)$ is shown. (*a*) High-speed streak with two low-speed streaks on the side during the calm phase at time $t = 2500\ \delta^*/U_\infty$. (*b*) Strong quasi-streamwise vortices which lean over the active low-speed streak at $t = 3100\ \delta^*/U_\infty$. (*c*) Breakdown at $t = 3300\ \delta^*/U_\infty$. (*d*) Initial structures regenerated with a shift in the spanwise direction at $t = 3470\ \delta^*/U_\infty$.

As the domain length is slowly decreased, the periodicity of the edge state is lost. The resulting dynamics on the edge is chaotic, though also consisting of calm and bursting regions. The space-time diagram corresponding to one of the edge trajectories in this case is shown in figure 1(*b*). The structures remain localised and the bursts still correspond to shifts in the spanwise direction. However, the direction and the distance of those shifts is no longer fixed but varies in an unpredictable fashion. Slowly lowering the box length we find intermediate states which may suggest the character of bifurcations at play that lead from ordered and periodic towards chaotic edge states.

References

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