AN RNG BASED K-EPSILON TURBULENCE MODEL USING THE REALIZABLE EDDY VISCOSITY FORMULATION

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Abstract The three popular high-Re k-ε models are Standard k-ε, RNG k-ε and Realizable k-ε. The RNG k-ε has an additional term in the ε equation, compared to the Standard k-ε, which is mainly responsible for the difference in performance between the two models. Similarly, the Realizable k-ε has a different model dissipation rate equation and eddy viscosity formulation. In the present study, an attempt is made to incorporate the performance enhancing feature of each model into a single model and thus formulate the Hybrid k-ε turbulence model. Subsequently, this model is implemented in an in-house finite element based Navier-Stokes solver and tested for benchmark flows. The Hybrid k-ε model seems to be an improvement over the existing k-ε models.

K-EPSILON MODEL

The k-ε model is a simple and complete two equation Reynolds-averaged Navier-Stokes based model which has become popular mainly because it is computationally inexpensive. Several low-Re and comparatively fewer high-Re variants of the k-ε model are available today but the former are not as computationally inexpensive as the latter. Thus the idea of using high-Re k-ε models in conjunction with wall functions still remains attractive. The Standard k-ε model was the first to be introduced but did not perform well for certain classes of flows. The RNG k-ε is a mathematically derived k-ε model, which is almost similar to the Standard k-ε except for an additional term in the ε equation, which is ad hoc and not derived from RNG theory [1]. The difference in performance between the Standard k-ε and RNG k-ε models is attributed to this ad hoc term. Moreover, the model constants of RNG k-ε are mathematically derived and not empirically adjusted like in the Standard k-ε. Additionally, one of the assumptions taken for deriving the model constants of RNG k-ε is that the value of C_μ is constant and equal to 0.0845 [2]. The Realizable k-ε has a different model dissipation rate equation and eddy viscosity formulation. The realizability in the Realizable k-ε is enforced by the eddy viscosity formulation, according to which C_μ is not a constant instead is related to the mean strain rate [3]. In the present study, the Hybrid k-ε turbulence model is formulated which is similar to the mathematically derived RNG k-ε but with C_μ being defined by the realizable eddy viscosity formulation. The values of the model constants are revised using the current formulation of C_μ.

FINITE ELEMENT METHOD

To test the accuracy of a model it is imperative that errors from all other sources should be kept to a minimum. Since the artificial diffusion in the solutions, obtained through finite element analysis of high-Re flows, is much lower compared to those obtained through finite volume method, finite element method is ideal for testing the accuracy of a model. However it is well known that when Galerkin Finite Element Method (GFEM) is used for performing numerical simulations of advection dominated flows, it produces oscillatory solutions. To overcome this one has to use stabilized versions of GFEM such as Streamline Upwind Petrov-Galerkin (SUPG) FEM or Galerkin Least Squares (GLS) FEM. In the present study an in-house Navier-Stokes solver based on SUPG-FEM, proposed by Brooks and Hughes [4], is employed to perform the numerical simulations.

PLANE ASYMMETRIC DIFFUSER

The plane asymmetric diffuser is one of the benchmark problems on which the Hybrid k-ε is tested. Studies were carried on the plane asymmetric diffuser by Buice and Eaton [5]. The Reynolds number ($Re_{H,cl}$) of the flow based on inlet channel height (H) and the centreline velocity at the throat of the diffuser is 20,000. The flow in the diffuser is subjected to an adverse pressure gradient and also undergoes separation and reattachment. Unlike the backward facing step, the point of separation is not so obvious in the case of the asymmetric diffuser within which the flow undergoes pressure driven separation from a smooth wall. Studies have shown that the point of reattachment can only be predicted correctly if the point of separation is known to some level of accuracy thus the asymmetric diffuser makes it more challenging for the turbulence models to predict the point of reattachment.

The stream-wise velocity profiles, obtained using the Standard, RNG, Realizable and Hybrid k-ε, at various cross sections along the asymmetric diffuser and its comparison with the experimental results are shown in Figure 1. It is observed that the Hybrid k-ε gives improved predictions in the case of the plane asymmetric diffuser. This performance evaluation is extended to include other benchmark flows such as channel flow, backward facing step, flow past a circular cylinder, etc.

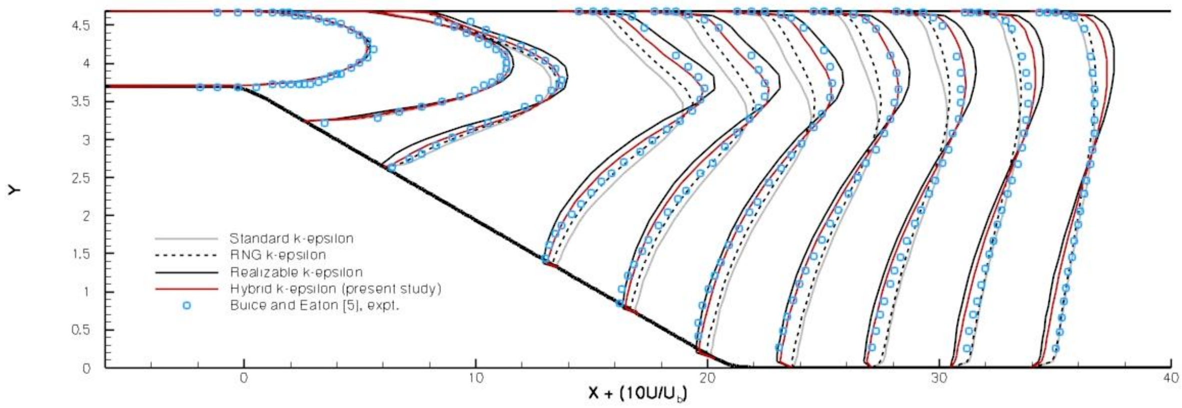

Figure 1. Comparison of the stream-wise velocity profiles.

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