STREAMLINE GEOMETRY AND THE TURBULENT DISSIPATION FIELD

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Abstract The highly intermittent nature of the instantaneous dissipation field is analyzed based on highly resolved direct numerical simulations of homogeneous isotropic turbulence at various Taylor based Reynolds numbers. The dissipation field is decomposed into the composition stemming from the enstrophy ω^2 and the one from the second invariant of the velocity gradient tensor Q. The latter seems to regulate the lower intermittency of the dissipation field ε compared to the intermittency of the enstrophy field as it often times counteracts large positive bursts of ω^2. Based on a local coordinate system attached to streamlines, we decompose Q into three different contributions which are amenable to a geometric interpretation in terms of streamlines and related geometries. It turns out that the term proportional to the local curvature of streamlines and the gradient of the turbulent kinetic energy perpendicular to the streamline regulates the large bursts of the enstrophy.

ANALYSIS

The instantaneous enstrophy (vorticity squared) field and that of the instantaneous dissipation are known to be closely related,

$$\varepsilon = 2\nu s_{ij}s_{ij} = \nu\left(\omega^2 - 4Q\right), \tag{1}$$

where s_{ij} denotes the symmetric part of the velocity gradient tensor, ω^2 the enstrophy and Q the second invariant of the velocity gradient tensor. Simple averaging shows that in homogeneous isotropic turbulence, as $\langle Q\rangle = 0$, the mean dissipation is directly proportional to the mean enstrophy $\langle\varepsilon\rangle = \nu\langle\omega^2\rangle$. However, this average relation does not imply anything about the instantaneous spatial correlations of the dissipation field and the enstrophy field. In fact, until recently it was believed that the enstrophy field is more intermittent than the dissipation field, cf. [4, 2]. However, recent analyses based on highly resolved DNS covering a range of Reynolds numbers up to $Re_\lambda = 1000$ imply that at high Reynolds numbers the two statistics seem to approach each other, cf. [7]. For the understanding of the intermittent structure of the instantaneous dissipation field in turbulent flows it is thus necessary to better understand the interplay of the enstrophy field with Q. This work focuses on the role of the Q field and how it influences the intermittent character of the dissipation field. To this end we propose to decompose Q into terms which are amenable to a geometric interpretation in a natural coordinate system attached to streamlines based on the fluctuating velocity field. We denote with t_i the unit tangent vector to streamlines

$$t_i = \frac{dx_i}{ds} = \frac{u_i}{u}. \tag{2}$$

The unit normal vector n_i points in direction of the "acceleration" along the curve

$$n_i = \frac{1}{\kappa}\frac{dt_i}{ds}, \tag{3}$$

where $\kappa = |dt_i/ds|$ denotes the geometric curvature of streamlines. The system is completed with the binormal vector defined as

$$\vec{r} = \vec{t}\times\vec{n}. \tag{4}$$

Then, the rate of change of all three vectors is described by Frenet's formulas [1]. The geometric properties of particle paths (the analogon to streamlines in an evolving turbulent field) have for instance been studied by Rao [5] and Braun et al. [1], whose ideas have been extended to the geometric properties of streamlines by Schaefer [6]. With the above definitions we can express Q as

$$Q = -\left[\underbrace{k\left(H^2 - K\right)}_{T_1} + \underbrace{\kappa\frac{\partial k}{\partial n}}_{T_2}\right], \tag{5}$$

where $k = u^2/2$ denotes the turbulent kinetic energy field, H and K denote the two invariants of the streamline curvature tensor $\partial t_i/\partial x_j$ and $\partial/\partial n$ the gradient projected in n_i direction perpendicular to the streamline. The streamline curvature tensor $\partial t_i/\partial x_j$ and thus its invariants encode geometric information of the infinitesimal surface locally normal to a streamline, cf. [3], while the curvature κ describes the geometry of the streamline itself.

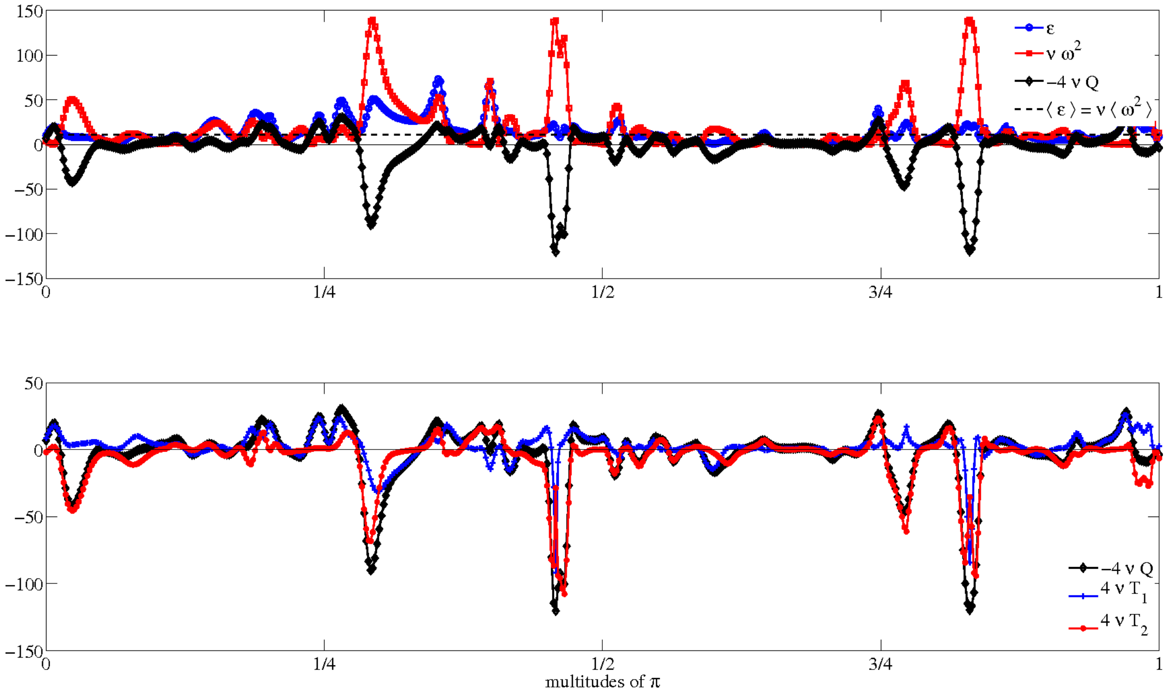

Figure 1. Top: instantaneous balance of the three terms in eq. (1). Bottom: instantaneous balance of the three terms in eq. (5).

RESULTS

Figure 1 (top) shows a representative instantaneous realization of the three terms in eq. (1) along an arbitrary coordinate direction in the DNS of homogeneous isotropic forced turbulence with a Taylor based Reynolds number of $Re_\lambda = 180$. The dotted line indicates the mean energy of the entire box. One clearly observes the intermittent structure of the dissipation as well as the enstrophy field with the latter being more intermittent than the first. One especially observes that the term proportional to Q "regulates" the largest peaks of the enstrophy field yielding a less intermittent dissipation field. On the other hand large values of the dissipation seem to preferably occur in regions where Q is close to zero or negative so that the enstrophy is not "damped" or even amplified.

To better understand the dynamics of Q, figure 1 (bottom) shows the decomposition of the term proportional to Q following eq. (5). It is clearly observable that the large positive peaks of Q lead to a dampening of the large peaks of the enstrophy and are mainly due to the term labeled T_2 in eq. (5) which is proportional to the geometric curvature of the streamlines and the gradient of the turbulent kinetic energy perpendicular to the local streamline direction. On the other hand the comparably quite negative fluctuations of this term seem to be dominated by the first term labeled T_1.

Further statistical analysis will be carried out based on four different DNS of homogeneous isotropic turbulence with Taylor based Reynolds numbers in the range of $Re_\lambda = 100 - 350$. This will allow to assess the Reynolds number dependence of the different terms in the decomposition of Q and its relation to the dissipation and enstrophy field. Also, the decomposition in eq. (5) will be used to further explain the role of (local and non-local) pressure fluctuations induced by large values of Q based on the Poisson equation for incompressible turbulence.

References

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