WAVES FROM A SUPERSONICALLY-REVOLVING DISTURBANCE IN RELATION TO OVER-REFLECTION INSTABILITY OF AN ANNULAR FLOW: IN-LAB SHALLOW-WATER SIMULATIONS

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Abstract Presented is an experimental study on geometry and kinematics of the ‘shock’ waves exited by a disturbance revolving in shallow water. The results are used for analyzing and interpreting such details of earlier model experiments on the over-reflection instability of an annular supersonic flow as a self-organization mechanism, 2D features and kinematic characteristics of the wave structures generated by the instability.

Wave generation due to curvilinear supersonic motion of a disturbance came first to the attention of researchers in practical and theoretical aeronautics because of the possibility of useful non-trivial behaviour of the sonic-boom fronts from smoothly-turning jet planes. Having passed some distance to the centre of curvature of the trajectory, the ‘inner’ one of the two fronts could suffer a sharp kink, which looked like a kind of reflection, and thence went backwards, gradually receding from the centre to the trajectory [1, 2]. Later these aeronautical data were not verified in a complex theoretical study based on Lighthill's kinematic wave method in application to the general case of dispersive media and combined with hydrodynamic experiments with soap films [3, 4]. This time the front was shown to experience not reversion but termination in its point closest to the centre.

Meanwhile the phenomenon can play an important role in dynamics of non-straight supersonic flows. If the wave from such a disturbance on a streamline in a boundary or shear zone of the flow does return back to the same streamline, it can trigger generation of noise or even destabilization of the flow due to the mechanism of wave over-reflection [5–7]. Processes of the type can be the key elements in developing the over-reflection instability of differentially rotating compressible media [8, 9], which is of remarkable significance for understanding dynamics of astrophysical discs [10].

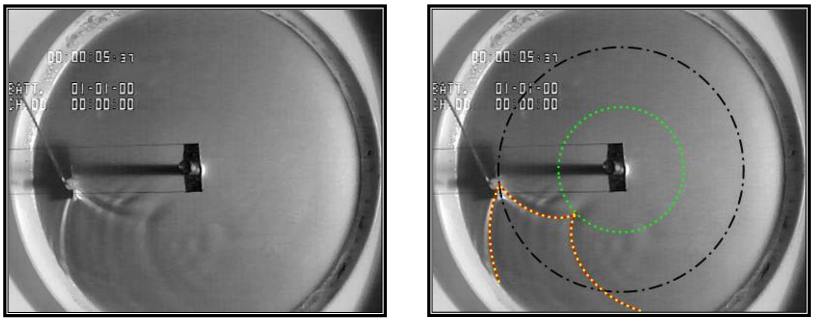

Figure 1. Shallow-water waves from a disturbance dipped into a thin layer of liquid and revolving in it ‘supersonically’ – experiment (**left**) and its comparison with theory (**right**). **Dot-and-dash line** – orbit of the disturbance, **green dots** – circular evolute, **yellow-red line** – circle involute. Here and later on all revolutions and rotations are clockwise.

Beyond the practical and theoretical aeronautics, the existence of the front reversion has been first confirmed in the presented laboratory simulations with shallow-water waves in a thin free-surface layer of liquid as a model of sound in 2D gas. The waves from a small body (artificial disturbance), which is dipped into the layer and revolves there in a circular orbit, do behave exactly like the fronts from the supersonic planes. The loci of the waves observed (Huygens-Mach fronts) turn out to be excellently described both qualitatively and quantitatively by the equations of a circle involute. The radius of the corresponding circular evolute equals the linear velocity of the (‘sonic’) shallow-water waves divided by the angular velocity of the disturbance. The evolute itself is like a ‘guideway’ for the reversion point that moves along the circle at the sonic speed. Among unordinary features of the phenomenon emerged in the simulations, there is wave generation by the disturbance at the sonic motion of the latter. In this peculiar case, when the evolute coincides with the orbit, the inner wave degrades into nothing, but the wave outside the orbit remains to be generated, appearing well-pronounced and full in amplitude. Nothing of the kind could obviously happen if the disturbance moved rectilinearly.

The results of the simulations have been used for analysing and interpreting data of the earlier experiments on the over-reflection instability of an annular (zonal) ‘supersonic’ flow in shallow water [9]. The flow was externally maintained around a circular ‘core’, i.e. around an initially-still central part of a free-surface liquid layer. Due to the instability, coherent wave structures were exited in the core, varying in their order of axial symmetry (mode number *m*) and frequency of rotation. The structures can be considered as resonant superpositions of the above kind of waves reflected multiply from the edge of the core (from the inner boundary of the flow). In this approach, the core acts as an acoustic resonator independent, in principle, on whether the waves are amplified at the reflection or are not. Parameters of the eigenmodes calculated for such a resonator correlate very well with those of the wave structures observed in the experiments (design, mode numbers, and rotation frequencies).

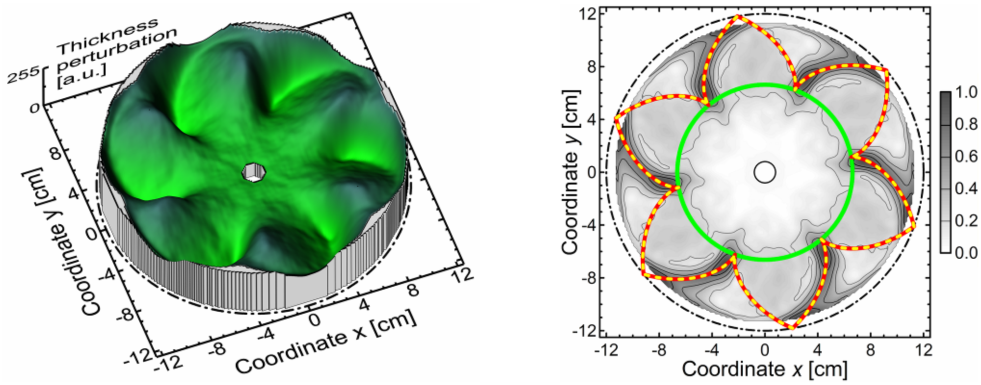

Figure 2. On the left: thickness perturbation of the liquid layer in a wave structure generated by the mode $m = 6$ of the over-reflection instability of an annular ‘supersonic’ flow in the model experiments with shallow water. **On the right**: gradient magnitude of the same thickness perturbation overlaid with a ‘resonant’ chain of six involute segments. **Dot-and-dash lines** – inner boundary of the flow and orbit of the disturbance, **green line** – circular evolute, **yellow-red lines** – segments of the circle involute.

Also this simple approach allows one to clarify easily the origin of some significant features that were theoretically and experimentally found to be typical of the structures but not explained within the general numerical study [8, 9]. Among them, there are increase in the perturbation amplitude with approaching to the evolute and absence of any perturbation behind the evolute in the very central area of the core.

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