STABILITY OF COUNTERFLOWING SUPERFLUID HE-II UNDER UNIFORM MUTUAL FRICTION

S Arun , R Sooraj & A Sameen
Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai, India

Abstract Modal and non-modal stability analysis of counterflowing superfluid He-II in a channel is attempted in this paper. Landau's two fluid model is assumed and the mutual friction between two components is assumed to be uniform across the channel. Mutual friction is found to be stabilizing normal fluid when a linear modal and non-modal stability analysis were performed. When both the components are perturbed simultaneously, though linear analysis shows the flow is stable, the transient growth shows instability even for Reynolds number as lows as 1

INTRODUCTION

Helium has two isotopes, of which ^{4}He has very low viscosity below 2.17 K and referred as superfluid Helium or He II. The hydrodynamics of superfluid is explained using Landau's two fluid model. The model describes superfluid as an inseparable mixture of two fluids, a viscous normal fluid component and an inviscid superfluid component, each having different velocity and density fields. The interaction between two components is represented using a mutual friction term, which is proportional to the relative velocity of two components. Stability analysis of plane Poiseuille flow of He II, in which both the components flow in same direction, has been attempted before. Godfrey *et al*[1] and Bergström[2] found mutual friction destabilizes normal fluid, where as Sooraj & Sameen[3] modified the mean profile as Reynolds number dependent and showed that mutual friction stabilizes He II. Here we consider thermally excited counterflow turbulence in a channel. In counterflow turbulence, experimentally it was observed that the vortex line density across the channel is uniform[4]. For counterflowing superfluid, Galantucci *et al*[5] assumed uniform mutual friction across the channel and computed superfluid velocity profile when normal fluid velocity profile is parabolic. The same profiles are used here to investigate the effect of uniform mutual friction on stability of the flow. Two scenarios are investigated; one in which only the normal component is perturbed and the other in which both the components are perturbed simultaneously.

FORMULATION

The normal fluid and superfluid momentum equations are coupled through mutual friction term, which arises due to interaction of normal fluid with the quantized vortices in superfluid. The mutual friction, which depends on temperature and relative density of superfluid component, is simplified as [1]

$$F_{mf} = f(y)\,(V_n - V_s)\,, \tag{1}$$

where V_n and V_s are normal and superfluid velocities and $f(y)$ depends on the superfluid vortex line density across the channel. A parabolic normal fluid velocity is assumed, for which Galantucci *et al*[5] found the superfluid velocity also to be parabolic when the mutual friction is uniform across the channel. For the stability analysis we assume $V_s = 1/3 - y^2$ and $f(y)$ as constant. The Orr-Sommerfeld and Squire's equations for normal fluid component are as follows.

$$i\,(\alpha U_n - \omega)\left(D^2 - k^2\right)\hat{v}_n - i\alpha U_n''\hat{v}_n = \frac{1}{Re}\left(D^2 - k^2\right) - f\left(D^2 - k^2\right)\hat{v}_n + f\left(D^2 - k^2\right)\hat{v}_s \tag{2}$$

$$\left[i\,(\alpha U_n - \omega) - \frac{1}{Re}\left(D^2 - k^2\right) + f\right]\hat{\eta}_n = -i\beta U_n'\hat{v}_n + f\hat{\eta}_s \tag{3}$$

$\tilde{v}_n = \hat{v}_n(y)e^{i(\alpha x+\beta z-\omega t)}$ and $\tilde{\eta}_n = \hat{\eta}_n(y)e^{i(\alpha x+\beta z-\omega t)}$ are the normal fluid velocity and vorticity perturbations respectively. α and β are streamwise and spanwise wavenumbers and ω is frequency of perturbation wave. D represents differentiation in y and $k^2 = \alpha^2 + \beta^2$. $\tilde{v}_s$ and $\tilde{\eta}_s$ are superfluid velocity and vorticity perturbations with the same wavenumbers and frequency as that of normal fluid. When the normal fluid component alone is subjected to disturbances, the terms containing $\tilde{v}_s$ and $\tilde{\eta}_s$ vanish.
The equations for wall normal velocity and vorticity perturbations for superfluid are;

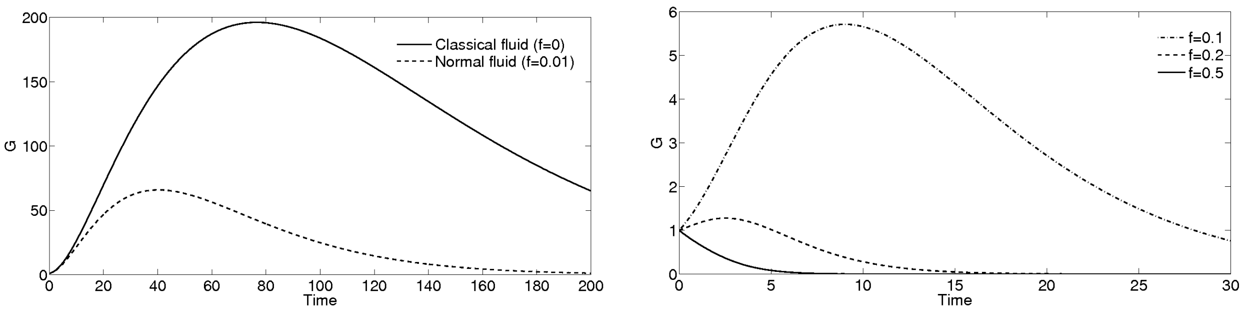
$$i\,(\alpha U_s - \omega)\left(D^2 - k^2\right)\hat{v}_s - i\alpha U_s''\hat{v}_s = -f\left(D^2 - k^2\right)\hat{v}_s + f\left(D^2 - k^2\right)\hat{v}_n \tag{4}$$

$$\left[i\,(\alpha U_s - \omega) + f\right]\hat{\eta}_s = -i\beta U_s'\hat{v}_s + f\hat{\eta}_n \tag{5}$$

These four equations [2-5] are simultaneously solved to predict the stability of the flow, subject to the boundary conditions $\hat{v}_n = D\hat{v}_n = \hat{v}_s = \hat{\eta}_n = \hat{\eta}_s = 0$ at the walls, using Chebyshev spectral methods.

MODAL AND NON-MODAL STABILITY

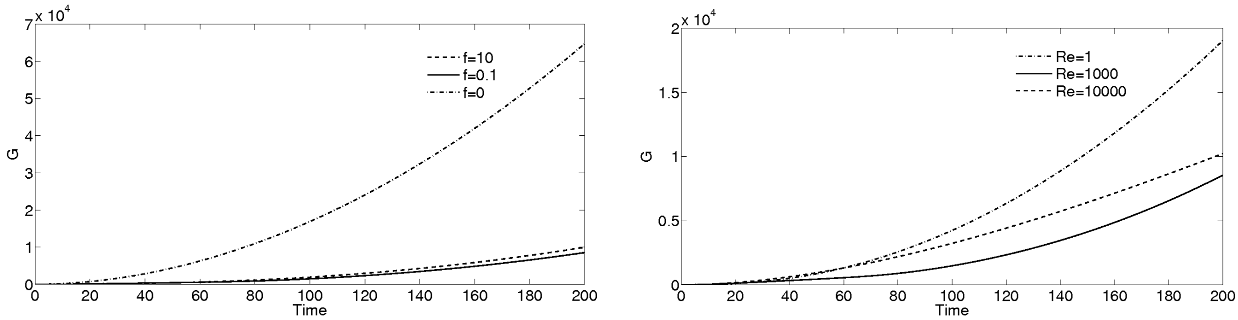
For a uniform mutual friction across the channel, normal fluid is found to be exponentially stable without any unstable modes for Reynolds number as high as 10^5. When both components are simultaneously perturbed, it is found that almost all the modes obtained are neutrally stable. The results of linear stability analysis shows that mutual friction stabilizes the normal fluid.
The transient growth analysis, similar to as that in [3, 6], for normal fluid also underlines the same. Even for a very small value of mutual friction $(f = 0.01)$, the growth is only about 30% as that observed in classical fluids (fig.1(a)). At high values of f, the damping is more evident. From fig.1(b), for $f = 0.5$ the initial perturbation is not allowed to grow and is damped monotonically even at a Reynolds number where the classical fluid tends to become unstable.



(a) (b)

Figure 1. Transient growth in normal fluid $\alpha = 0$, $\beta = 2$ **(a)** $Re = 1000$. **(b)** $Re = 8000$

However, non-modal analysis, when both the components are simultaneously disturbed, predicts a different result. Though the flow is found to be stable even at very high Reynolds number in the modal analysis, the transient growth makes the flow highly unstable (fig.2(a)). The instability is seen at very low Reynolds number (fig.2(b)) and even in the absence of mutual friction coupling. Transient growth observed in this case can be attributed to the interaction between normal fluid modes and superfluid modes which is not through mutual friction, but due to non-orthogonality of modes. (More will be discussed about this at the conference).



(a) (b)

Figure 2. Transient growth when both components are simultaneously perturbed $(\alpha = 0, \beta = 2)$. **(a)** At $Re = 1000$ for different values of f, **(b)** For $f = 0.1$ at different Reynolds number

References

[1] S. P. Godfrey, D. C. Samuels, and C. F. Barenghi, "Linear stability of laminar plane Poiseuille flow of Helium II under a nonuniform mutual friction forcing," *Phys. Fluids*, vol. 13, no. 4, p. 983, 2001.

[2] L. B. Bergström, "The initial-value problem for three-dimensional disturbances in plane Poiseuille flow of Helium II," *J. Fluid Mech.*, vol. 598, pp. 227–244, 2008.

[3] R. Sooraj and A. Sameen, "Effect of vortex line distribution in superfluid plane Poiseuille flow instability," *J. Fluid Mech.*, (accepted),2013.

[4] L. Skrbek and K. R. Sreenivasan, "Developed quantum turbulence and its decay," *Phys. Fluids*, vol. 24, p. 011301, 2012.

[5] L. Galantucci, C. F. Barenghi, M.Sciacca, M.Quadrio, and P.Luchini, "Turbulent superfluid profiles in a counterflow channel," *J. Low Temp. Phy.*, vol. 162, pp. 354–360, 2011.

[6] P. J. Schmid and D. S. Henningson, *Stability and Transition in Shear Flows.* Springer, 2001.