MANIFESTATIONS OF NON-UNIVERSAL NATURE IN DECAYING MHD TURBULENCE

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Abstract Evidence for lack of universality in freely decaying MHD turbulence was reported only two years ago [10]. Theoretical support for these recent results do not exist yet. Therefore, we try to gain insight into this problem by combining, to our knowledge, the highest resolution simulations ever performed for freely decaying isotropic MHD turbulence (2048^3) with theoretical analysis to investigate the non-universal nature of such flows. We focus on the origins of non-universal statistics that are hidden in the turbulent cascades by studying the topology of the flow fields and the geometric statistics of various vector quantities.

INTRODUCTION

The classical phenomenology of Kolmogorov (K41) [8] conjectures that, in a regime of intermediate scales, the dynamics are minimally affected by forcing, boundaries and large-scale anisotropies (which are generally flow-dependent), and unaffected by the viscous dissipative effects that occur at the very small scales. The dynamics in this so-called inertial range are dominated by the nonlinear term of the Navier-Stokes equations, and it seems reasonable under assumptions of homogeneity and isotropy that inertial range dynamics should display statistically universal behaviour. This is also directly related to the assumption of local interactions, i.e. interactions between comparable scales. Several attempts have been done to extend K41 for hydrodynamic turbulence to MHD [7, 9, 6, 15, 4]. However, numerous studies have shown that the key assumptions of small-scale universality, isotropy and locality of interactions are in question in various MHD contexts [1, 2, 14]. Besides intermittency corrections, K41 gives to a good approximation the power law of the energy spectrum in hydrodynamic case in contrast to MHD, where several debatable phenomenological theories exist [15, 11]. This has many implications as the energy dissipation rate, which is required to predict for example heating rates in solar and space physics, depends on the slope of the energy spectrum. Moreover, subgrid scale models, required for numerical modelling in astro- and geo-physics, are less developed in MHD as a result of the lack of detailed knowledge of its energy spectrum.

In freely decaying isotropic MHD turbulence, some simulations obtained Iroshnikov-Kraichnan (IK) scaling while others K41 scaling [13, 12, 11]. Recently, large resolution simulations by Lee et al. [10] (using a code that enforced some symmetries to achieve higher resolution) demonstrated dependence on initial conditions and thereby lack of universality of freely decaying MHD turbulent flows. However, the afore mentioned studies were performed either with moderate resolution [11] or under the assumptions of symmetry preservation [10]. To have clear evidence of non-universality in MHD turbulence all the symmetries need to be broken and the Reynolds number should be sufficiently large for power laws to be clearly demonstrated.

In this study, we perform numerical simulations on massively parallel computer clusters with resolution up to 2048^3. To our knowledge, a 2048^3 resolution without imposing any symmetries has not been achieved yet in isotropic MHD turbulence simulations. Isotropic in the sence that there is no magnetic flux either in or out of our periodic boxes. Therefore, we have a unique opportunity to test our theories on well separated scale regimes in order to understand the basis behind the non-universal nature of these fluids carrying a magnetic field.

ANALYSIS AND RESULTS

We analyse numerical computations of decaying MHD turbulence with random initial conditions but also the three different cases studied by Lee et al. [10]. In detail, Lee et al. considered three runs with the Taylor-Green (TG) vortex for the initial velocity field and three different initial magnetic fields which are generalisations of the TG vortex symmetries to MHD [10]. These initial conditions have the same ideal invariants, i.e. total energy, zero magnetic helicity and comparable cross-helicity (between 0 and 4% in relative terms when normalised with the energy). Moreover, the fields are normalised such that the kinetic and magnetic energies are in equipartition at time zero. Finally, there is no uniform magnetic field and the magnetic Prandtl number is always equal to unity. According to Lee et al. [10] the evolution of the ratio of magnetic to kinetic energy distinguishes the three cases (see Fig. 1(a)), which end up with three different scaling exponents for the energy spectra at the peak of dissipation (see Fig. 1(b)). Lee et al. [10] argued that depending on the correlations of the initial fields, energy spectra compatible with Kolmogorov $k^{-5/3}$, Iroshnikov-Kraichnan $k^{-3/2}$, and weak turbulence theory k^{-2} may emerge (Fig. 1(b)).

Fig. 1 is essentially a reproduction of the results by Lee et al. [10] obtained by our simulations for 1024^3 resolution. However, note that our runs were evolved without imposing any symmetry constrains unlike in [10], allowing thus the

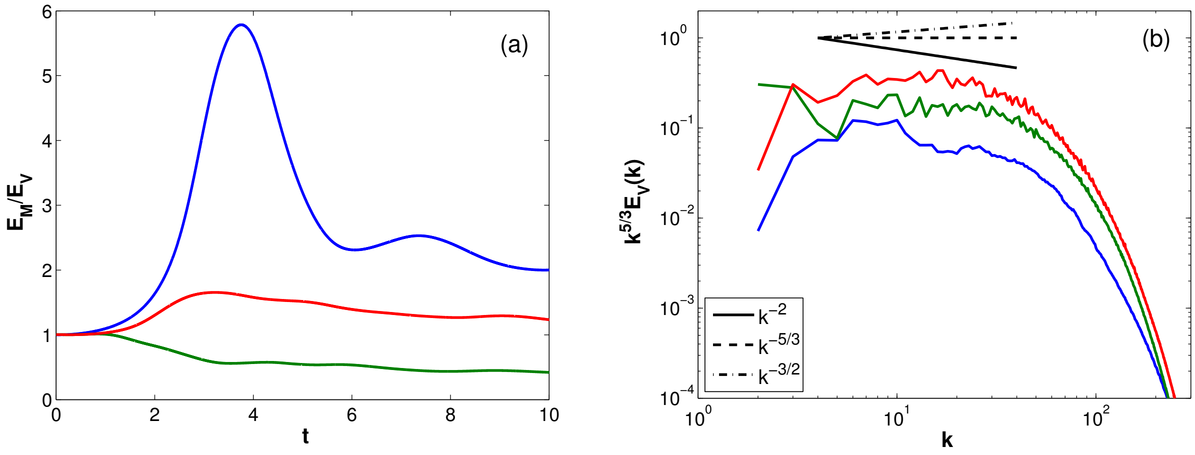

Figure 1. (a) Temporal evolution of the magnetic to kinetic energy ratio. (b) Kinetic energy spectra compensated by $k^{5/3}$ at the peak of dissipation. The blue, red and green lines represent the cases with the three different initial conditions for the magnetic field. The resolution of these runs is 1024^3.

turbulence to evolve freely with the hope that the initial TG symmetries will break at high enough Reynolds numbers. However, even for our 1024^3 runs the TG symmetries are not broken. They seem to be a strong property of the MHD equations that is preserved by time evolution of the solutions. So, questions we attempt to address in this study are:

- What is the role of the symmetries imposed by the initial conditions?
- What happens if we break these symmetries? Will the scaling of the energy spectra change?
- Do we have classes of universality for these moderate Reynolds numbers or is there a universal power law in the high Reynolds number limit?

Therefore, we add perturbations of different amplitude in the initial configurations to investigate the time it takes for the symmetries to break compared to the time to reach the maximum dissipation and we check if the exponent of the energy spectrum is altered due to the symmetry breaking.

Interestingly, it is argued [10] that the k^{-2} energy spectrum that we also obtain in one of the Lee et al. cases is due to weak turbulent effects. However, recent results [5] reveal that this exponent is rather due to the structures created, reminiscent to structures in Burgers turbulence [3], influenced by the initial conditions of the velocity and the magnetic fields. Since little information can be extracted just from the energy spectra, our approach was to gain insight on the origins of non-universal statistics by studying the topology of the fields as well as alignments of various vector quantities.

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