EFFECTS OF NUMERICAL ERRORS IN LARGE-EDDY SIMULATION OF TRANSITIONAL WALL-BOUNDED FLOWS

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Abstract Effects of numerical errors in large-eddy simulations of transitional wall-bounded flows are investigated. Temporal and spatial laminar-to-turbulent transition in both channel and pipe flows is simulated using second- and fourth-order accurate finite-volume solvers based on the open-source CFD package OpenFOAM. Starting from unperturbed laminar or plug-flow profiles, it is found that, for centered finite-volume discretization, the onset of primary instability and the subsequent nonlinear pattern are highly sensitive to rounding errors and depend on a multitude of parameters: grid resolution, initial base flow, number of processors, flux reconstruction scheme, temporal integration time-step and subgrid-scale model. An attempt aimed at identifying the different contributions coming from each parameter, the various sources of error and their influence on the transition scenario is given in the paper. Although the results confirm the potential of large-eddy simulation for reflecting the true nature of fluid dynamics, they also emphasize that great care should be exercised when using this technique for transitional flows, especially in applications of practical engineering importance.

INTRODUCTION

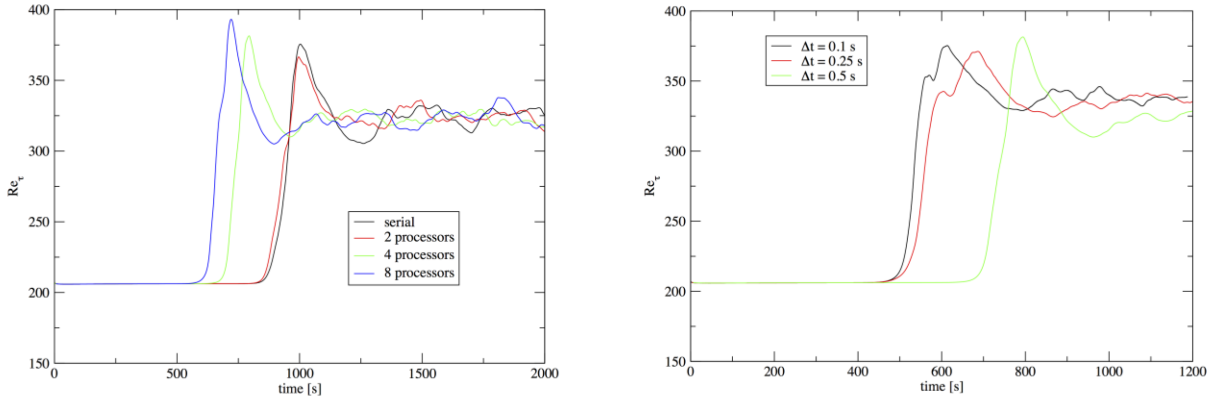
Large-eddy simulation (LES) has become a powerful and reliable tool to predict a broad range of turbulent flows [1]. Its inherent ability to explicitly resolve large-scale unsteady motions allows one to capture the physics of fluid flows at reasonable computational cost and moderate modeling effort. On the other hand, it is well-known that LES suffers from severe numerical requirements, as the computational grid is usually fine enough to resolve the energy-carrying scales, which have not to be contaminated nor dissipated by rounding, truncation and aliasing errors [2]. Control of numerical error is particularly crucial for transitional flows, in which very small disturbances may eventually evolve into relevant turbulent fluctuations, as a result of remarkably complex interactions between the base flow and various instability modes that must be resolved appropriately [3]. If the numerical method does not provide a sufficiently low noise-to-signal ratio, as in the case of unstructured finite-volume solvers commonly used in engineering, the simulation can be severely polluted or even dominated by numerical noise. This is unacceptable for many flows of engineering interest in which laminar-turbulent transition plays a critical role, and where a reliable prediction of transition is essential [4]. Furthermore, these complex flows are influenced by a large number of factors, and the effect of numerical errors on transition might be indistinguishable from other simulation parameters, resulting in a difficult, if not impossible, verification and validation process.

Although the influence of numerical errors in large-eddy simulation has been well established for turbulent flows, both in terms of truncation and aliasing effects [5-6], application of LES to transitional flows has become an active field of research only recently [7-8] and a comprehensive framework for the influence of numerical errors on transition has not yet been defined. Moreover, effects of rounding errors and spurious numerical waves are often under-estimated, especially in conjunction with transition. In this regard, it has recently been demonstrated that rounding errors and spurious waves can be a severe source of unpredictability for large-eddy simulations, by: 1) inducing separation of trajectories for fully turbulent flows - especially in massively parallel computations [9-10]; 2) triggering hydrodynamic modes and eventually leading to fictitious breakdown for transitional flows [11].

In the present work, spatial and temporal transition in two prototype wall-bounded flows, plane channel and pipe, is investigated. Such classical, well-documented flows allow one to control and distinguish the influence of the various parameters on the transition process. The selected computational solver is based on the open-source CFD package OpenFOAM, which is provided with a well-tested and validated LES capability for canonical problems [12-13]. The incompressible flow equations are solved by means of an unstructured, collocated finite-volume solver with a segregated PISO algorithm. For convective fluxes, 2^{nd} order centered and upwind and 4^{th} order centered reconstructions are used; for time-integration, a 2^{nd} order backward scheme is adopted. Boundary conditions are periodic along one or two homogeneous directions for temporal or spatial simulations, respectively; no-slip at the walls is enforced. No disturbances are super-imposed onto the initial field, and transition is triggered by numerical errors. A systematic comparison of LES results for different grid resolutions, initial base flow (plug-flow or laminar profile), number of processors, flux reconstruction scheme, temporal integration time-step and subgrid-scale model is performed. The different contributions and sources of error, as well as their influence on the transition process, are discussed; the concept of critical Reynolds number is finally assessed and reviewed in light of numerical results.

PRELIMINARY RESULTS

In the following figures, some preliminary results for temporal transition are presented for a pipe flow, $Re_\tau = 360$ (same case of [14]); an immediate, though integral**,** indication for transition is the friction Reynolds number Re_τ as a function of time; see caption for details on the simulations. In Figure 1a, the effect of the parallel-computing configuration is shown. Present simulations show that the onset of transition anticipates as the number of processors is increased, while the subsequent pattern, including the typical overshoot, is qualitatively the same; the resulting fully turbulent field is also – in a statistical sense – invariant. This can be explained in terms of rounding and commutation errors at parallel interfaces, which result in an increasingly strong random forcing for the Navier-Stoke equations [9]. In contrast, the effect of time-step (Figure 1b), which inherently comprises dissipation and dispersion characteristics [15-16], not only influences the onset of transition, but also the overshoot and the resulting fully developed turbulent state. In both cases, after secondary instabilities, the skin friction is underestimated with respect to the nominal value of $Re_\tau = 360$ [14]: no subgrid-scale models have been used in this case, hence no sufficient shear stress is provided.



(a) *(b)*

Figure 1. Variation of friction Reynolds number as a function of time in a pipe at nominal $Re_\tau = 360$, using a central 2nd order scheme, a coarse mesh (60000 cells) and no SGS-model. *(a)* Effect of parallel-computing configuration, $\Delta t = 0.5$s. *(b)* Effect of Δt, 4 processors.

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