DRAG REDUCTION IN TURBULENT CHANNEL FLOWS USING RANS MODEL

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Abstract **This work presents results for drag reducing (DR) flows in turbulent plan channel flow using a new eddy viscosity model coupled with the kinetic k and dissipation ε equations and the RANS (Reynolds Averaged Navier-Stokes) equations. The model is based on a viscous theory for polymer drag reduction in which elastic polymers would act as an additional effective viscosity, growing linearly with the distance from the wall in the buffer layer**.

THE MODIFIED GOVERNING EQUATIONS

The use of elastic polymers as drag reducing agents (DRA) for turbulent channel flows is an interesting research topic for the turbulence community and has many industrial and practical applications. L'vov et al. [1], and De Angelis et al. [2] using direct numerical simulation (DNS) of Navier-Stokes equations, suggest that in the presence of few ppm of elastic polymers diluted in a turbulent flow, the effective viscosity near the wall region, can be provided by a linear profile, resulting in a modified Navier-Stokes equation, assuming that $\nu_0 \to \nu(y)$ and $p \to P$, given by

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu(y)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right], \quad i = 1,2 \tag{1}$$

where

$$P = p + \frac{\partial \nu(y)}{\partial t} + u_j \frac{\partial \nu(y)}{\partial x_j}, \tag{2}$$

and

$$\nu(y) = \nu_0 + \nu_p \mathcal{R}, \tag{3}$$

with ν_p being the polymeric viscosity and $\mathcal{R}$ is the average dimensionless extension tensor. In order to obtain the formulation for the RANS model, we apply the Reynolds decomposition ($u_i = U_i + u_i^{'}$ and $P = \overline{p} + p'$) in momentum equation (1). Using the Boussinesq relationship ($\overline{u_i^{'} u_j^{'}} = -2\nu_T D_{ij} + \frac{2}{3}\kappa\delta_{ij}$), the modified equation time-averaged for the mean flow field is give by:

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{\partial}{\partial x_i}\overline{p_e} + \frac{\partial \nu(y)}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \nu(y)\frac{\partial}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}\right) + \frac{\partial (2\nu_T D_{ij})}{\partial x_j}, \tag{4}$$

where $\overline{p_e} = \overline{p} + \frac{2}{3}\kappa$, is the effective scalar pressure field and κ is the turbulent kinetic energy, and D_{ij} the rate-of-deformation tensor.

We are interested in describing a linear profile of the viscosity, $\nu(y)$, for channel flows. For this objective, it will be necessary to investigate the dimensionless value of the slope, denoted by C, for the piecewise linear profile along the channel height. Detailed analysis of the influence of the slope on the drag reduction calculation will be reported. As shown by the numerical results, the slope C is an important parameter to obtain real drag reduction levels as observed in the experiments and recent numerical direct simulations [4].

The procedure used to derivate the new equations for the turbulence transport model is the same used for the classical κ-ε model. The main modification is the application of the momentum equation (1) to derive the modified turbulence transport equations. These new equations for k and ϵ are :

$$\frac{\partial \kappa}{\partial t} + \frac{\partial (U_j \kappa)}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left[\left(\nu(y) + \frac{\nu_T}{\sigma_\kappa}\right)\frac{\partial \kappa}{\partial x_j}\right] - \varepsilon + \frac{\partial \nu(y)}{\partial x_j}\frac{\partial \overline{u_i^{'} u_j^{'}}}{\partial x_i}, \tag{5}$$

and

$$\begin{aligned} \frac{\partial \varepsilon}{\partial t} &+ \frac{\partial (U_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu(y) + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}P_k - C_{2\varepsilon}\frac{\varepsilon^2}{k}\varepsilon \\ &+ \frac{1}{\nu(y)}\frac{\partial \nu(y)}{\partial x_j}\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j} + U_j\frac{\varepsilon}{\nu(y)}\frac{\partial \nu(y)}{\partial x_j} + \frac{\varepsilon}{\nu(y)}\left(\frac{\partial \nu(y)}{\partial x_j}\right)^2 - 3\frac{\partial \nu(y)}{\partial x_j}\frac{\partial \varepsilon}{\partial x_j}, \end{aligned} \tag{6}$$

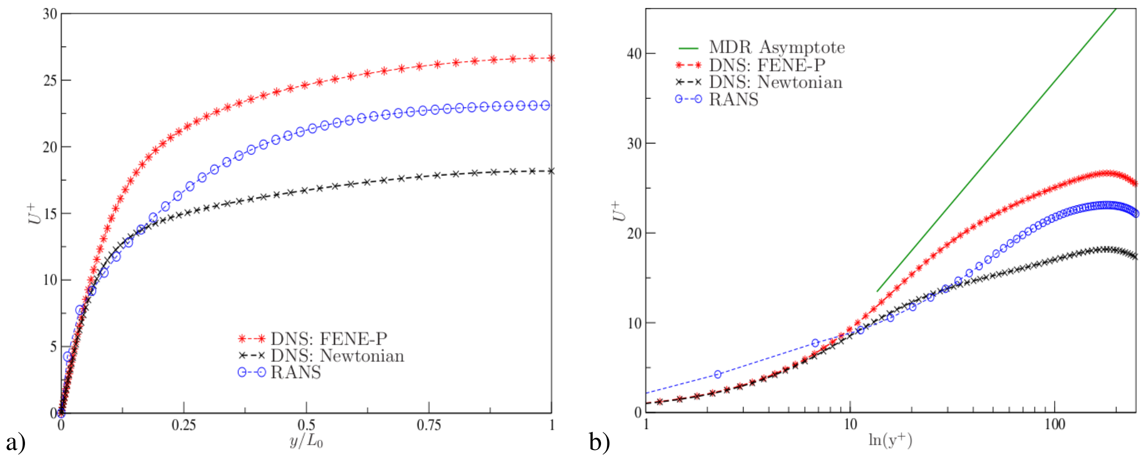

Figure 1. Results for RANS model with a linear viscosity profile using $C = 20$ and DNS result [4]: a) mean velocity distributions across channel half-width and b) comparison between numerical predictions and the theoretical universal MDR asymptote.

where $P_k = -\overline{u'_i u'_j}\frac{\partial U_i}{\partial x_j}$ is the turbulent production, $\nu_T = C_\mu \frac{\kappa^2}{\varepsilon}$ is the turbulent viscosity and ε is the turbulent dissipation rate. In addition, we consider the following model constants: $C_\mu = 0.09$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$, $\sigma_\kappa = 1.0$ and $\sigma_\varepsilon = 1.3$.

NUMERICAL RESULTS

The mass conservation and the modified Navier-Stokes (4) equations are solved using the explicit version of the code proposed by Oishi et al. [3], which combines projection methods with a Marker-And-Cell discretization. The main modifications to the solution algorithm adopted in this work consist in the implementation of the modified transport equations (4), (5) and (6). The simulations were conducted in a rectangular channel of half-height L_0 and length $40L_0$. The following data were used in the simulation: length scale of the flow $L_0 = 1$ m; characteristic velocity scale $U_0 = 1.0\text{ms}^{-1}$; kinematic viscosity of the fluid $\nu_0 = 0.00035714\text{m}^2\text{s}^{-1}$, corresponding to Reynolds number $Re = 2800$. To study the convergence of the numerical method, the channel flow was simulated on two meshes: M1 ($\delta x = \delta y = 0.05$, 40×800-cells) and M2 ($\delta x = \delta y = 0.025$, 80×1600-cells). The percentage of the drag reduction obtained in our RANS simulations for different values of the normalized slope C are presented in Table 1. We can observe levels of drag reduction up to 35.6 % and see that as C increases the DR level also increases. The drag reduction was calculated as $DR\% = \frac{\tau^N_{\text{wall}} - \tau^P_{\text{wall}}}{\tau^N_{\text{wall}}} \times 100$ where $\tau_{\text{wall}} = \frac{1}{Re}\frac{\partial u}{\partial y}\Big|_{\text{wall}}$.

Table 1. Drag reduction for the RANS model with a linear viscosity profile for different values of the slope C and Re_τ.

Slope (C) & Re_τ	0&233	8&227	10&223	12&217	14&210	16&203	18&195	20&189
$DR\%$ for M1 mesh	—	4.10	8.16	12.3	17.6	23.3	29.9	33.3
$DR\%$ for M2 mesh	—	4.73	8.51	12.8	18.1	24.3	30.4	35.6

In addition, Fig. 1 shows comparitive results between our RANS formulation using a linear viscosity profile with $C = 20$ and the DNS data obtained by Thais et al. [4]. From this figure, we can see that the velocity profile, given by the RANS model, is showing the right trend when drag reduction is acting in near wall turbulent flows.

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References

[1] V.S. L'vov, A. Pomyalov, I. Procaccia, and V. Tiberkevich. Drag reduction by polymers in wall bounded turbulence, *Phys. Rev. E* **92**: 1–4, 2004.
[2] E. De Angelis, C. M. Casciola, V. S. L'vov, A. Pomyalov, I. Procaccia, and V. Tiberkevich. Drag reduction by a linear viscosity profile. *Phys. Rev. E* **70**: 1–4, 2004.
[3] C.M. Oishi, F.P. Martins, M.F. Tomé, J.A. Cuminato and S. McKee, Numerical Solution of the eXtended Pom-Pom model for viscoelastic free surface flows, *J. Non-Newton. Fluid Mech.* **166**: 165-179, 2011.
[4] L. Thais, T. B. Gatski, and G. Mompean, Some dynamical features of the turbulent flow of a viscoelastic fluid for reduced drag, *J. of Turbulence* **13**: 1-26, 2012.