# Lagrangian attractors pattern for heavy particle in Kinematic Simulation

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**Abstract** The clustering of heavy particles is studied using the Kinematic Simulation approach. This synthetic model consists of a pre-determined Eulerian flow field given as an analytical formula. The flow periodicity is enforced by an arithmetic wave number distribution, and a Kolmogorov energy spectrum is used as an input. The heavy particles are distributed uniformly in the periodic box. The Lagrangian tracking of the particles is made accounting for the effect of gravity (drift parameter 'y') and inertia (Stokes number 'St'). Particles are allowed to settle with time until the asymptotic set of positions does not evolve further. The different shapes of this set or `Lagrangian attractor' are analysed. For this study, the Stokes number is chosen  $St \le I$ . A 3-dimensional Box Counting method is used for estimating the fractal dimension of the particles' attractor. It is noted that the attractor's dimension varies from one dimensional to three dimensional and is a function of the different pairs of St and  $\gamma$ .

### KINEMATIC SIMULATED TURBULENCE MODEL

Kinematic Simulation (KS) is a synthetic model of turbulence. The Eulerian flow field is generated using Fourier modes [1] and then the flow is integrated to obtain the particles positions and trajectory. The particles are considered as passive scalar in the flow. The equation for the flow field is given as:

$$u(x,t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} a_{ijl} \, Cos(K_{ijl},x) + b_{ijl} \, Sin(k_{ijl},x)$$
(1)

Where N and  $K_{ijl}$  represent the total number of Fourier modes and wave vectors defined by decomposition coefficients  $a_{ijl}$  and  $b_{iil}$ . Rather than the usual random orientation for the wave vectors of the Fourier series, an arithmetic decomposition is applied to keep the flow periodic. A Kolmogorov spectrum is used as the energy input for the flow.

## HEAVY PARTICLES IN THE TURBULENT FLOW

Heavy particles behave differently from fluid particles and the study of heavy particles in the flow is of significant importance in relation to many practical applications such as the homogenous mixing of fuel in direct ignition engines, transport mechanisms, heat transfer and rain formation etc.

The equation of motion of particles may have body and external forces such as gravity, buoyancy, added mass term etc. But for the present work, the simplified equation of motion (2) is taken into account for the Lagrangian tracking of particles;

$$\frac{dv}{dt} = \frac{1}{\tau_a} \left( u - V - V_d \right) \tag{2}$$

Where 'V' and 'u' are respectively the velocity of the fluid and of the inertial particle.  $V_d = \tau_{ag}$  represents the particle's drift velocity. The effect of two non-dimensional parameters: the Stokes number 'St' and the drift parameter ' $\gamma$ ', on the clustering of inertial particles is investigated.

$$St = \frac{\tau_a}{L/u'} \tag{3}$$

$$\gamma = \frac{V_d}{u'}$$

*u*' is rms of fluid velocity.

### **RESULTS AND DISCUSSION**

Previous studies have shown that inertial particles are found to settle in the turbulent flow at preferential locations [2] and clustering of inertial particles possessed different fractal patterns [3]. In the present work, the settling patterns of the particles have different topological dimensions as illustrated in Fig. 1 depending on the non–dimensional parameters Stokes number and drift parameter.



Fig.1 Particles' final set of locations with St=0.413 (a)  $\gamma$ =0.621, (b)  $\gamma$ =0.586 and (c)  $\gamma$ =5.745.

We focus on quantifying the particles set pattern dependency on *St* and  $\gamma$ . The box counting method is employed to characterise the set formed by the particles location. Iso-values for this `dimension' are plotted in Fig. 2. They describe the dimension and final topology of the set of particles for different pairs of *St* and  $\gamma$ .



Fig. 2 Contour plot showing the particles' attractor dimension as a function of St and  $\gamma$ 

# References

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