SCALE-SIMILARITY MODEL FOR STUDY OF FORCED COMPRESSIBLE MAGNETOHYDRODYNAMIC TURBULENCE

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Abstract We investigate scale-similarity model for study of forced compressible magnetohydrodynamic turbulence. The scale-similarity model has several important advantages in contrast to the eddy viscosity subgrid closures. It is commonly accepted that the scale-similarity model reproduces the correlation between actual and model turbulent stress tensor very well even when the flow is highly anisotropic. Moreover, it does not require a determining of special model constants in contrast to the eddy-viscosity closures. Therefore, it alone does not dissipate sufficiently energy and usually leads to inaccurate results in decaying cases of magnetohydrodynamic turbulence. However, the situation changes significantly when a forced magnetohydrodynamic turbulence is considered. Numerical computations under various similarity parameters are carried out and the obtained results are analyzed by means of comparison with results of direct numerical simulation and Smagorinsky closure for magnetohydrodynamics. It is shown that the scale-similarity model provides good accuracy and the results agree well with the direct numerical simulation results.

INTRODUCTION

Large-eddy simulation (LES) method is conquering more and more new areas of application for example magnetohydrodynamics (MHD). A study of various problems of MHD both in a space, astrophysical, helio- or geophysical flows and in applied industry problem requires large-scale modeling of the real physical conditions. Development of efficient numerical techniques and up-to-date computer systems allows to perform realistic MHD simulation of the turbulent flows. These simulations are extremely important for understanding the complex physics especially when object is beyond the reach of direct experimental study. Note that the MHD problems differ from those of the neutral fluid hydrodynamics. The MHD equations contain two fields, which introduces considerably more freedom into the dynamics, for instance the presence of both direct and inverse spectral cascades. Also, there are self-organization processes in MHD turbulence that have no hydrodynamic counterpart, namely, conservation of cross-helicity leads to highly correlated, or aligned, states, while conservation of magnetic helicity gives rise to the formation of force-free magnetic configurations. In general, new applications of LES put old questions concerning the use of subgrid-scale (SGS) models. There is no guarantee that the results obtained for hydrodynamics neutral fluid can be directly extended to the case of MHD in consequence of the above-mentioned distinctions. Determination of the optimal SGS parameterizations is a separate non-trivial task especially for forced MHD turbulence. Moreover, LES is not as well established in MHD turbulence as it is in hydrodynamics because the physics is more complex. Experimental verification of the validity of a subgrid-scale model and the calibration of model parameters are very difficult, since MHD turbulence is mainly observed in astrophysical systems and not in controllable laboratory experiments. It is therefore desirable to have subgrid closure without model constants since it is impossible to obtain model constants from in situ measurements in many application MHD turbulence in problems of aerospace industry and astrophysics. It is worth noting that the problem become more complicated when the compressible fluid is considered. The effect of compressibility on the structure of turbulence is an important and difficult topic in turbulence modeling. In order to solve such problems, LES approach is a useful and perspective method. The present study demonstrates that the scale-similarity model for forced MHD turbulence can be used as a stand alone SGS model as opposed to decaying case. The scale similarity parametrization has evident advantages the main ones being to reproduce rightly the correlation between the tensors between actual and model turbulent stress tensor for isotropic flow as well as for anisotropic fluid flow, and the absence of special model constants in contrast to other SGS closures. However, the scale-similarity model does not dissipate energy enough and usually leads to inaccurate results in decaying turbulence. But the situation changes substantially when a forced turbulence is considered. In this case, subgrid modeling in LES approach has to provide correct stationary regime of the turbulence rather than to guarantee proper energy dissipation. Therefore, in the present work LES method for modeling of compressible forced MHD turbulence is applied for various similarity parameters.

NUMERICAL RESULTS

Driving forces in our calculations were determined by linear forcing theory. Linear forcing algorithm is applied to keep the characteristics of turbulence stationary in time. Linear forcing is very fast because there is no need to transform from a spectral to physical space. The idea behind is that writing the transport equation for fluctuation velocity, the production term is proportional to the fluctuation velocity. The coefficient is the indicator of the balance between the production and dissipation. Since compressible MHD turbulence is considered in the present paper, the system of MHD equations includes the magnetic induction equation, and in this case the driving force is proportional to the magnetic field in the induction equation. Numerical computations were carried out and the obtained results were analyzed by means of comparison with results of direct numerical simulation.

We consider several numerical cases varying the similarity parameters. For the first case similarity numbers are: $Re \approx 300$, $Rem \approx 50$, $Ms \approx 0.35$, $\gamma = 1.5$. An important criterion for evaluating the quality of LES subgrid models is the correct time evolution of scalar quantities that characterize the global state of the simulated system. Results of computations of of *Urms* show that both SGS models reproduce correctly temporal evolution of root-mean-square velocity. The transient period of *Urms* is short because of the choice of initial conditions. DNS results have oscillations and LES results are displayed accurately these oscillations. The Smagorinsky model is more dissipative SGS closure than the scale-similarity model as expected. However, both SGS models demonstrate good agreement with DNS results. Time evolution of root-mean-square magnetic field *Brms* show that the transient period of magnetic field is longer than for velocity. Both SGS parameterizations achieve a stationary regime correctly and properly, but the scale-similarity model shows more precise coincidence with DNS. Time dynamics of the cross-helicity demonstrates that initially Smagorinsky model is more accurate but then scale-similarity closure provides better agreement with DNS results. All results for time dynamics of mean density fluctuate around the mean value after initial period. The investigation of inertial range properties is one of the main tasks in studies of scale-similarity spectra of MHD turbulence. Inertial range properties are defined as time averages over periods of stationary turbulence conditions. Total energy is the sum of kinetic and magnetic energy $ET = EM + EK$. It is worth noting that the famous spectra of Iroshnikov-Kraichnan and Kolmogorov-Obukhov for MHD turbulence were obtained for the total energy. As can be seen from results of our computations, the scale-similarity model yields more accurate results than Smagorinsky parametrization for MHD case, that is, results of the scale-similarity model produce better conformance with DNS results. There is well-defined inertial Kolmogorov-like range of $k{-}5/3$. Thus, subgrid-scale models with linear forcing can correctly reproduce the scale-invariant properties of the MHD turbulent flow. Time evolution of the kinetic and magnetic energy subgrid scale dissipation along with velocity molecular dissipation and magnetic molecular dissipation of turbulent field are analysed. The magnetic energy subgrid-scale dissipation is less for scale-similarity model while Smagorinsky closure is more dissipative model. Therefore, scale-similarity model is more accurate SGS parametrization for time evolution of *brms* but Smagorinsky model turns out excessively dissipative one. The second case of compressible MHD turbulence considered corresponds the case when the similarity numbers are: $Re \approx 650$, $Rem \approx 20$, $Ms \approx 0.23$, $\gamma = 1.5$. In initial transient period, DNS results of *Urms* drop very fast and then able to recover and to achieve a stationary average value. The transient period is longer for the second case than for the first case and the values of *Urms* oscillate weaker. The both SGS models produce adequate results and show good agreement with DNS results in stationary regime. From the time evolution of we can see that initially magnetic field increases rapidly and then attains statistically stationary level. The DNS results achieve this stationary level faster than SGS parameterizations. The scale-similarity model provides more accurately results then the Smagorinsky model for compressible MHD turbulence. The Smagorinsly model is more dissipative SGS closure. Notice that the scale-similarity model for MHD is more precise one as well as in the first case. Initially, there is large discrepancy in the results for mean density of the SGS models and the DNS results, but upon reaching steady state, these differences diminish. For the spectrum of the total energy the results of both SGS models are close to one another but scale-similarity model is slightly more accurate model. The third case corresponds to the numerical computations when the similarity numbers are: $Re \approx 1000$, $Rem \approx 220$, $Ms \approx 0.50.$, $\gamma = 1.5$. For the root-mean-square velocity *Urms* as a function of time both SGS models reproduce well the velocity fluctuations with time. Differences between the results of DNS and subgrid scale parameterizations are observed at the initial time interval and then all the results are very close. Note that the extended Smagorinsky model as well as the scale-similarity model for compressible MHD turbulence gives almost identical results for *Urms*. For time dynamics of the root-mean-square magnetic field *Brms* in the transient period of time, the results of DNS grow faster than the results of subgrid-scale models. As previously discussed in the numerical cases, the Smagorinsky model is more dissipative than the scale similarity one for the the magnetic field and the Smagorinsky model results are worse in agreement with DNS. For the third case, the scale-similarity model provides more accurate results and these results agree reasonably well with the DNS results of *Brms* in the stationary regime of MHD turbulence.

It is shown that the scale-similarity model provides good accuracy and the results of this SGS model agree well with the DNS results. If differences between the results obtained by the scale-similarity model and the Smagorinsky closure for velocity field are insignificant, then the differences are considerable for magnetic field. For the magnetic field, discrepancies with the DNS results are substantially lower for scale-similarity model while the Smagorinsky parametrization for MHD is excessively dissipative. Thus, the scale-similarity model demonstrates more accurate results than the eddy-viscosity SGS model especially for the time evolution of the magnetic field. The obtained results show that the scale-similarity model might be a useful parametrization for simulating MHD turbulent system with driving forces and for study of scale-invariance properties of forced compressible MHD turbulence in the inertial range.