TURBULENT PAIR DISPERSION AT SHORT TIMES

Falkovich Gregory [1] , Frishman Anna [1]
[1] *Physics of Complex Systems, Weizmann Institute of Science, Rehovot Israel*

Abstract We develop an analytic formalism and derive new exact relations that express the short-time dispersion of fluid particles via the single-time velocity correlation functions in homogeneous isotropic and incompressible turbulence. The formalism establishes a bridge between single-time Eulerian and long-time Lagrangian pictures of turbulent flows. In particular, we derive an exact formula for a short-term counterpart of the long-time Richardson law, and we identify a conservation law of turbulent dispersion which is true even in non-stationary turbulence.

DESCRIPTION OF MAIN RESULTS

In this work we explore the short-time behavior of two Lagrangian tracers, initially separated by the vector $\vec{R}_0$, in a fully developed turbulent flow in d dimensions. We assume the turbulence to be incompressible, homogeneous, isotropic and stationary. We study the evolution of the correlation functions of the velocity difference, $\vec{u}(t)$, and the separation, $\vec{R}(t)$, between the particles.

We begin with the spatially smooth velocity, which is relevant beneath the dissipation scale, where $u = \sigma(t)R(t)$, with $\sigma(t)$ a random matrix. We show that if $\langle u^{b-m} R^{a-m} (u \cdot R)^m \rangle$ is an analytic function of the velocity, then it is statistically conserved in time for the choice $a + b = -d$ and arbitrary m. This result is a continuation to all times of the classical result of Furstenberg and Zeldovich that R^{-d} is a martingale at asymptotically long times. Unlike the latter, it holds even for non stationary turbulence.

For turbulence within the inertial range, where $\langle u^2 \rangle \propto R_0^{\xi_2}$, we suggest the generalization of the Furstenberg and Zeldovich moment, $\langle R(t)^{2-\xi_2-d} \rangle$, and show that while it is not a martingale, it is the only moment whose second time derivative is zero at $t = 0$. Using the Kolmogorov scaling $\xi_2 = \frac{2}{3}$, we find that $\langle (R(t)/R_0)^{-2/3} \rangle - 1 = -\frac{2}{27} |\bar{\epsilon}| t^3 R_0^{-2} + O(t^4)$ in 2d and $\langle (R(t)/R_0)^{-5/3} \rangle - 1 = \frac{14}{81} |\bar{\epsilon}| t^3 R_0^{-3-2/3} + O(t^4)$ in 3d. This result provides a direct manifestation of time reversal symmetry breaking for pair dispersion and one can view it as a counterpart to the Richardson law $\langle R(t)^2 \rangle \propto t^3$ for short times.