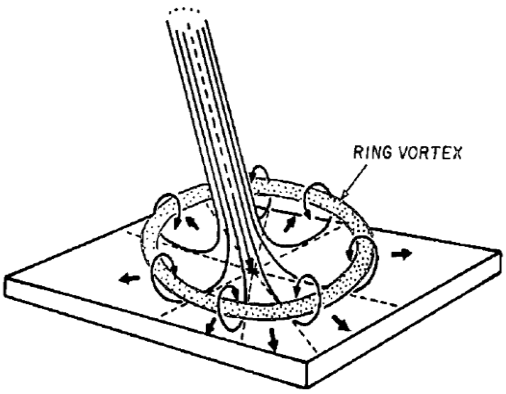
Vortex-Based Mixed Lagrangian-Eulerian Solver for Multi-Scale Fluid Dynamic Problem

Singh Nishant[1] & Taylor Ian[1]
[1]*Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow, UK*

Abstract This research will attempt to formulate a mixed Lagrangian-Eulerian vortex-based method as an alternative for large-scale large-eddy simulation (LES) for various applications, including acoustics and atmospheric simulations. Two cases of a simple expansion muffler (acoustics) and a downburst rig (atmospheric) are simulated as the reference simulations using LES with synthetic turbulence inflow boundary condition. The vortex-based Lagrangian-Eulerian method employs a laminar pressure-correction solver where turbulence is depicted by the Lagrangian particles and the vortex dynamics of these particles feedbacks to the Navier-Stokes equation for coupling the main flow with the turbulence. The simulation outcomes from the LES of muffler and downburst rig and initial results from the one way coupling of Lagrangian-Eulerian solver are presented. The mixed vortex-based method proposed in this work results in a ten-fold reduction in the simulation time.

Introduction

Many problems in fluid mechanics can be more conveniently described using vorticity and its evolution in time and space. This makes the vortex particle method and its characteristic of carrying information related to vorticity along with it, more important in such problems. Apart from that, the vortex methods usually require more points only where the flow has rotation and therefore computational expenses are relatively lower in comparison to the conventional finite volume methods. Vortex methods are also free from any dissipation related error as they do not model turbulence and only represents it in terms of vortex dynamics. Initial attempts of formulating vortex method include point vortex method and vortex blob method for two-dimensions [1]. Later on, Qian *et al.* [2] employed a cell-centered finite-volume method with integral form of vorticity transport equation. Further, Li *et al.* [3] extended it to three dimensions with a hexahedral grid system to solve three dimensional Navier-Stokes equations using finite volume method. Although these particle methods have provided good results for two dimensional simulations, their three dimensional simulation has not been equally successful. Selle *et al.* [4] in his work synergistically combined Lagrangian vortex particle method and Eulerian grid based method to overcome the weaknesses of both the methods. Recently, Pfaff *et al.* [5] have extended their work to introduce appropriate vortex particles to represent turbulence using artificial boundary layer and vortex interaction or transport.



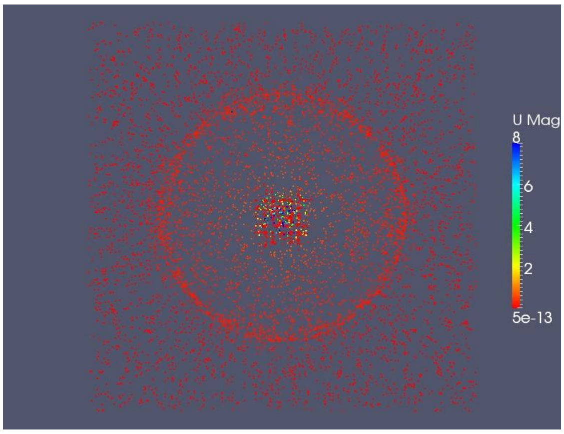


Figure 1: (a) Downburst sketch by Fujita [6] **(b)** Downburst simulation using mixed vortex method.

The presence of vortices in the problem domain creates length-scale related issues and the conventional finite element and finite volume methods usually would require very fine mesh to capture those scales. Most frequently used CFD methods like RANS have the inherent limitations due to associated averaging of the length-scales and modelling of the full spectrum instead of computing the broader spectrum. DNS can provide accurate results in separating flow cases but it comes at very high computational cost and is mostly not feasible for industrial applications. LES, on the other hand provides a very interesting opportunity for the CFD community to explore this area. However, the LES approach is found to require a turbulence representation at the inlet for an accurate simulation. Although an upstream flow condition can ideally provide a good inflow data, the computational boundary cannot, however, be extended upstream indefinitely. Even though these upstream flow methods or recycling methods are very accurate, these are computationally expensive and are not applicable to most of the flow simulations of practical importance. It is, therefore, preferable to use a lower order description provided by different related turbulent quantities. In some cases, random fluctuations are superimposed on uniform inlet velocity to achieve a turbulent inflow at the inlet without contaminating the inflow with upstream acoustic information. The generation of random velocity profile for inflow data to match actual turbulent flow field is a difficult and computationally challenging process. In this context, Synthetic boundary condition is found to be a good alternative for generation of inflow condition for LES. A mixed spectral method proposed by Singh [7] to generate a synthetic turbulent velocity field by extending the spectral method proposed by Davidson and Billson [8] has been adopted for LES of two different application areas of acoustic muffler and downburst rig.

A mixed Lagrangian-Eulerian vortex based method has been developed using particle vortex dynamics to represent turbulence characteristic with Lagrangian particles. The vortex seeding is done with a synthetic turbulence spectrum of Singh and the merging, breakup and dissipation of particles are considered in the light of classical turbulence theory. Two different applications of acoustic muffler and downburst rig (figure 1(a)) are selected for LES investigation and the case of downburst rig is selected for investigation using mixed vortex based method.

Simulation

A simple expansion muffler, with expansion chamber diameter to inlet/outlet pipe-length ratio of 1:14, is selected for preserving acoustic signals from contamination by unwanted reflections. LES is conducted with inlet flow of 10 *m/s* and Smagorinsky subgrid scale model is adopted. The velocity contour clearly shows the rolling up of vortices after the flow enters the inlet and the subsequent breakup of vortices after striking the rear wall of the pipe. The interaction of incoming acoustic signals and returning signals from the rear wall can clearly be observed in figure 2(a).

The downburst rig set up is using a 6*m*×6*m*×2*m* box with a one metre diameter inlet cylinder of length 1.5*m* entering the domain at the centre. A uniform mesh spacing of 25mm was used throughout the domain. An inlet velocity of 10 *m/s* was used with inlet that uses the mixed synthetic boundary condition. The mixed synthetic boundary condition showed a quicker transition to realistic turbulence. This is important when the inlet to floor distance is only 3.5*m* or 2*m* as turbulence does not have a long time to develop. The LES simulation was also more successful at capturing vortex generation, not only it captured the vortex generation near the expansion but also captured the smaller secondary vortices generated near the expansion and the bottom wall, illustrated in figure 2(b).

The mixed vortex-based method proposed in this work clearly reduces the time of simulation by ten folds. Initial result of simulation has been able to capture the vortex ring illustrated by Fujita, as shown in figure 1 (a) and (b).

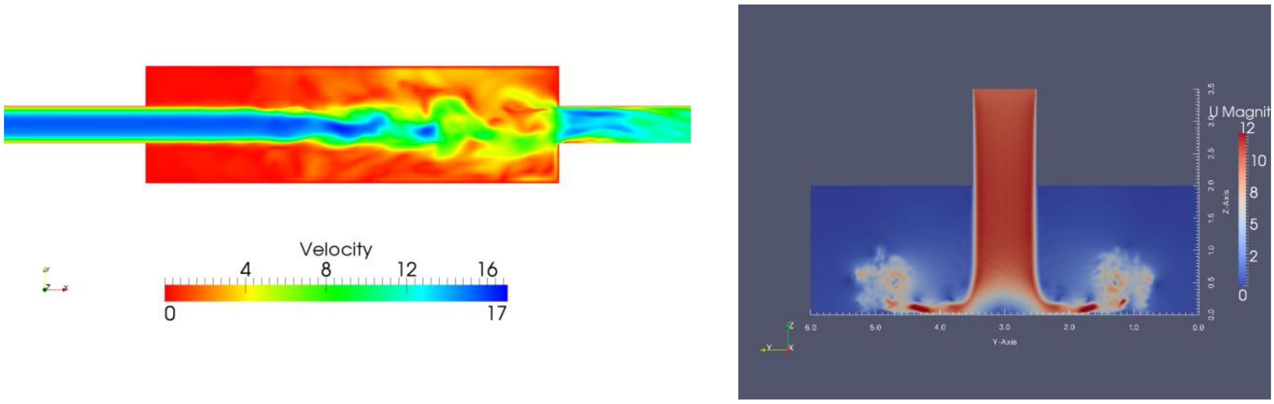


Figure 2: **(a)** Velocity contour of expansion muffler. **(b)** Velocity contour of downburst rig.

References

[1] Leonard, A. Vortex methods for flow simulation. *Journal of Computational Physics* **37**: 289-335, 1980.
[2] Qian, L. and Vezza, M. A vorticity-based method for incompressible unsteady viscous flows. *Jour. of Computational Physics* **172**: 515-542, 2001.
[3] Li, W. and Vezza, M. A hybrid vortex method for the simulation of three-dimensional flows. *Int. J. Numer. Meth. Fluids* **57**: 31-45, 2008.
[4] Selle, A., Rasmussen, N. and Fedkiw, R. A Vortex Particle Method for Smoke, Water and Explosions. *SIGGRAPH, ACM TOG* **24**, 910-914, 2005.
[5] Pfaff, T. Thuerey, N. Selle, A. and Gross, M. Synthetic turbulence using artificial boundary layers. *ACM Transactions on Graphics* **28** (5), 2009.
[6] Fujita, T.T., & Wakimoto, R.M. Five scales of airflow associated with a series of downbursts. *Monthly weather review*, **109**, 1439–1456, 1981.
[7] Singh, N.K. Large Eddy Simulation of Acoustic Propagation in Turbulent Flow through Ducts and Mufflers. *PhD thesis, University of Hull*, 2012.
[8] Davidson, l. and Dahlstrom, S. Hybrid LES-RANS: An approach to make LES applicable at high Reynolds no. *Int. J. CFD*, **19**: 415-427, 2007.