STUDY OF SHOCK/TURBULENCE INTERACTION USING OBSERVABLE EULER AND NAVIER STOKES EQUATION

Hareshram Natarajan[1] & Kamran Mohseni [1,2]

[1] *Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611, USA*

[2] *Department of Electrical and Computer Engineering, University of Florida, Gainesville, FL 32611, USA*

Abstract Problems involving shocks and turbulence in fluids are multi-scale in nature and are prone to continuous generation of high-wavenumber modes or small-scales. Viscous terms are generally introduced to remedy this high wave mode irregularity. An alternate regularization method was recently introduced [1] , dubbed as *observable* Euler and Navier-Stokes equations, by deriving the conservation laws based on *observable* divergence theorem. This study investigate the validity of these equations in simulating a series of turbulence and shock dominated flows. In particular, we will investigate the 2D shock vorticity/entropy wave interaction, compressible turbulence with shocklets, and 3D shock-turbulence interaction problem with observable Navier-Stokes equations and compare the results with available direct numerical simulations or LES.

Observable Equations: The non-linear term u.∇ u in the Euler equations leads to the evolution of higher wave modes as time evolves. Shocks and turbulence are the manifestation of this high wave mode irregularity. Classically, this equation is regularized with a dissipative term, such as viscosity or hyperviscosity. Here we propose an alternate inviscid regularization obtained by applying the observable divergence [1] to the Euler Equation. Recently, a series of publications [2, 3, 4] about observable divergence, observable Burgers equations and observable Euler equations demonstrate that using observable divergence to replace the nonlinear term is effective to generate the proper computational formulation for complicated flows. The observable divergence is $odivF = \overline{f}V + \overline{V} \cdot \nabla f$ where the vector field $F = fV$ and V is a vector. For any variable u, $\overline{u}$ is observable u, defined by a class of low pass filter $g : g * u = \overline{u}$. These filters need to reduce high frequency wave modes without affecting low frequency wave modes. They also have to satisfy: normalized($\int g = 1$), nonnegative($g(x) > 0, \forall x$), decreasing($\mid x_1 \mid \geq \mid x_2 \mid \Rightarrow g(x_1) \leq g(x_2)$), symmetric($\mid x_1 \mid = \mid x_2 \mid \Rightarrow g(x_1) = g(x_2)$) and fourier decay($\lim_{|k|\to\infty} \mid k \mid \widehat{g}(k) = 0$). In our computational analysis, we use the Helmhotz operator, which satisfies these conditions to generate observable quantities. The Helmholtz filter, $u = \overline{u} - \alpha^2 \Delta \overline{u}$, where α is the representative scale of the averaging.

When the Euler equations are re-derived based on the fluxes calculated from observable quantities, we obtain the observable Euler equations. Let V, ρ, e, E, p represent velocity, density, internal energy, total energy and pressure.
Then the observable Euler equations for compressible perfect gas are:

$$\rho_t + \overline{\rho} \nabla \cdot V + \overline{V} \cdot \nabla \rho = 0, \tag{1}$$

$$(\rho V)_t + \overline{\rho V} \nabla \cdot V + \overline{V} \cdot \nabla (\rho V) = -\nabla p \tag{2}$$

$$(\rho E)_t + \overline{(\rho E + p)} \nabla \cdot V + \overline{V} \cdot \nabla (\rho E + p) = \rho V \cdot \Sigma + s \tag{3}$$

where the total energy $E = \rho e + \frac{1}{2} \rho V \cdot V$, the pressure $p = (\gamma - 1)\rho e$, the temperature $T = (\gamma - 1)e/R$ and R is the gas constant, s is the external source of heat generation inside the domain and Σ is the sum of all external forces.

Problems considered. In this study we present the application of the observable Euler and Navier-Stokes equations to several problems in compressible flows. While our final submission will include simulation to 1D Shu-Osher problem, 2D shock-vorticity/entropy wave interaction, 3D compressible turbulence, and 3D shock-turbulence interaction, in this extended abstract we only present two sample simulations.

I.) 2D interaction of vorticity/entropy wave and a normal shock. The current problem has a incident vorticity-entropy wave interacting with a stationary shock. The ability of the code to capture both shocks and fluctuations simultaneously is tested. The 2D Observable Euler equations are solved in the domain $x_1 \in [0, 4\pi]$, $x_2 \in [-\pi, \pi]$, with $\Delta x_1 = \pi/50$ and $\Delta x_2 = \pi/16$ with α=0.1. The kinetic energy and vorticity amplification across the shock of $M = 1.5$ is studied for incident angles $\phi = 45^o$ and $\phi = 75^o$ and the results are compared with different numerical techniques in [5]. The results indicate that the observable equations are capable of capturing the shock wave interactions smoothly.

II.) 3D Compressible Turbulence with Eddy Shocklets. The case of decaying compressible turbulence with eddy shocklets is carried out. The 3D Observable Equations are solved in a domain $x_i \in [0, 2\pi)$ with $\Delta x = 2\pi/64$ and α =0.015. A sufficiently high turbulent Mach Number is given to develop eddy shocklets spontaneously. Evolution kinetic energy and enstrophy are plotted and the result are compared with reference solutions from [5]. The results show the Observable equations are able to capture the higher scales as well as the lower scales of the flow.

III.) 3D Shock/Turbulence Interaction. Interaction between isotropic turbulence and a normal shock of strength $M = 1.5$ is considered. The influence of vortical and entropy fluctuations on the interaction problem will be focused. The amplification of turbulent kinetic energy and transverse vorticity variances across the shock are studied and presented with the currently available DNS results [6] [7].

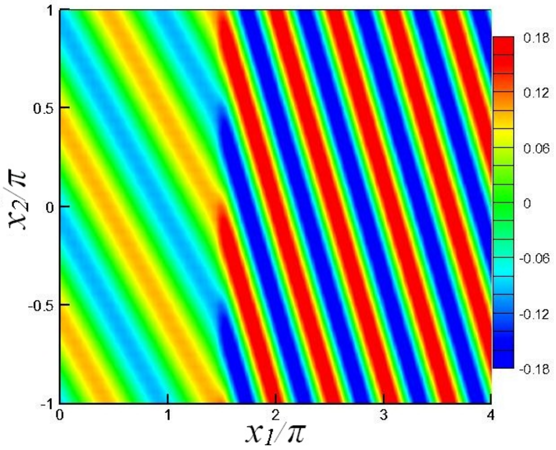

Figure 1. 2D shock-vorticity/entropy simulation at M=1.5, The instantaneous vorticity contour at t=25s for $\phi = 45^o$ and $k_1 = 1$.

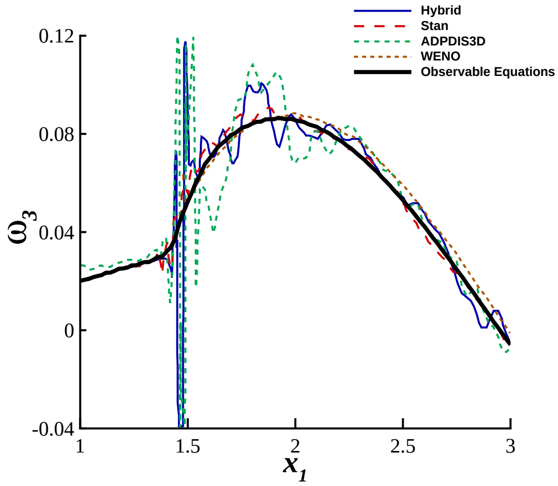

Figure 2. 2D shock-vorticity/entropy simulation at M=1.5, Instantaneous vorticity profile at $x_2 = 0$ for $\phi = 75^o$. Except the observable equation results, the rest of the curves are from [5].

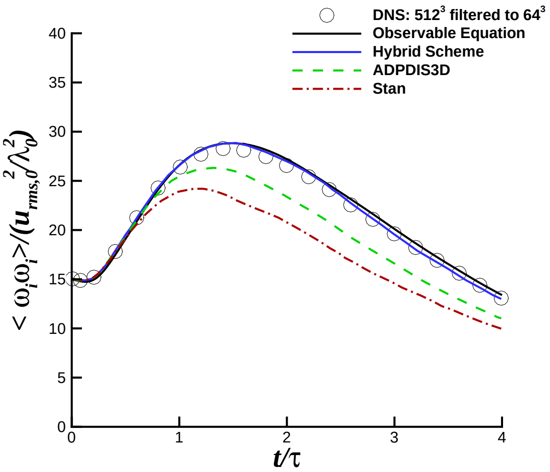

Figure 3. 3D compressible turbulence with eddy shocklets at $M_t = 0.6$, $Re_\lambda = 100$. The evolution of enstrophy compared with different schemes [5] for the 3D compressible turbulence with eddy shocklets.

References

[1] Mohseni, K., "Derivation and Numerical Simulation of Regularized (Observable) Euler Equations," AIAA paper 2010-1274, 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, Orlando, FL, January 4-7 2010.

[2] Norgard, G. and Mohseni, K., "On the Convergence of Convectively Filtered Burgers Equation to the Entropy Solution of Inviscid Burgers Equation," *SIAM Journal, Multiscale Modeling and Simulation.*, Vol. 7, No. 4, 2009, pp. 1811–1837.

[3] Norgard, G. and Mohseni, K., "An inviscid Regularization of the One-Dimensional Euler Equations," AIAA paper 2009-3567, 39th AIAA Fluid Dynamics Conference, San Antonio, Texas, June 22-25 2009.

[4] Norgard, G. and Mohseni, K., "A new potential regularization of the one-dimensional Euler and homentropic Euler equations," *SIAM Journal, Multiscale Modeling and Simulation.*, Vol. 8, No. 4, 2010, pp. 1212–1242.

[5] Johnsen, E., Larsson, J., Bhagatwala, A., Cabot, W., Moin, P., Olson, B., Rawat, P., Shankar, S., Sjgreen, B., Yee, H., Zhong, X., and Lele, S., "Assessment of high-resolution methods for numerical simulations of compressible turbulence with shock waves," *J. Comput. Phys.*, Vol. 229, 2010, pp. 1213–1237.

[6] Mahesh, K., Lele, S., and Moin, P., "The influence of entropy fluctuations on the interaction of turbulence with a shock wave," *J. Fluid Mech.*, Vol. 334,, 1997.

[7] Lee, S., Lele, S., and Moin, P., "Direct numerical simulation of isotropic turbulence interacting with a weak shock wave," *J. Fluid Mech.*, Vol. 251, 1993, pp. 533–62.