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ABSENCE OF TURBULENCE IN AN EXACT SOLUTION OF THE 3D NAVIER-STOKES EQUATION

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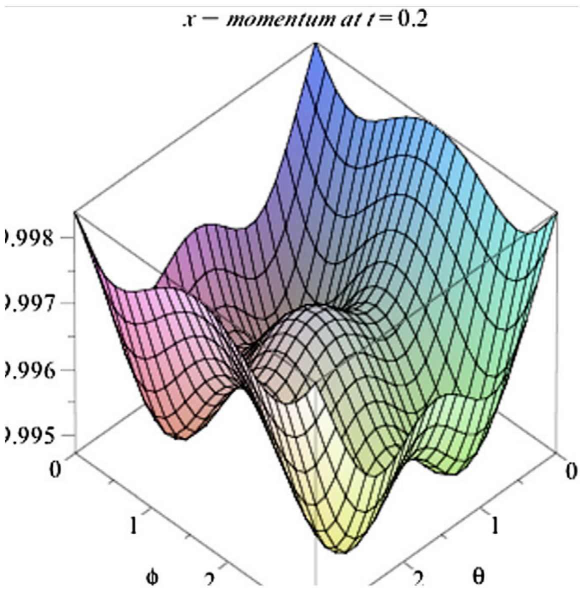
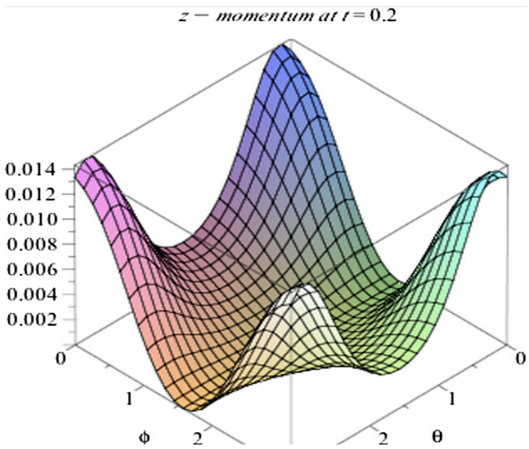
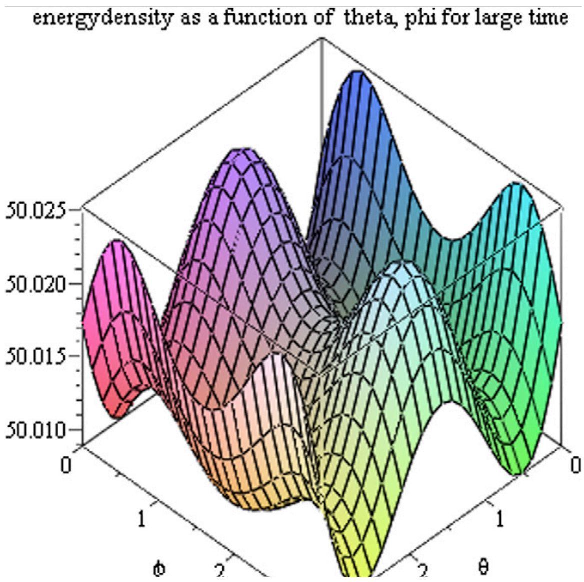
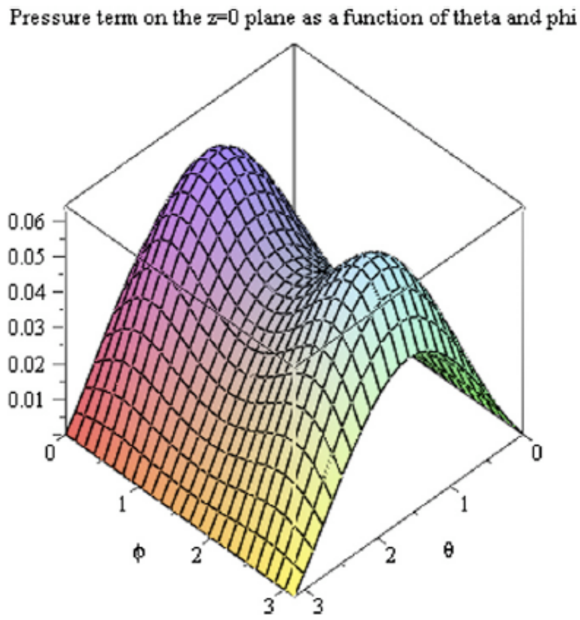
Abstract: A recent exact solution of the 3D Navier-Stokes aquation, the only exact solution published to date, shows a regular smooth solution for an initial one dimensional jet. The absence of what may be considered a turbulent solution raises questions about the origin of turbulence. In particular the Navier-Stokes equation is considered the problem definition of turbulence, if such solutions exist. The absence of a turbulent solution raises a fundamental issue on whether the Navier-Stokes equation hides turbulent solutions. With this result, we must open up the possibility that turbulence is to be explained using other kinetic or transport equations.

Reference [1] provides details for the analytic solutions for the Cartesian velocities, pressure and energy density inside a cube of dimension L. In this report we graphically display the velocities v_x, v_y, v_z, pressure and energy density for this cube with periodic boundary conditions. The plots shown are typical for an initial velocity in the x-direction. The y and z momenta are the same. The plots are made with the variable change $x = L\sin(\vartheta), y = L\sin(\varphi), z = L\sin(\omega)$. Increasing the initial x-velocity without limit does not yield any regime that may be characterized as turbulent. We thus raise the possibility that turbulence cannot ever be derived from the Navier-Stokes equation, and that it may well be that the Navier-Stokes equation is not the correct problem definition of turbulence [2].

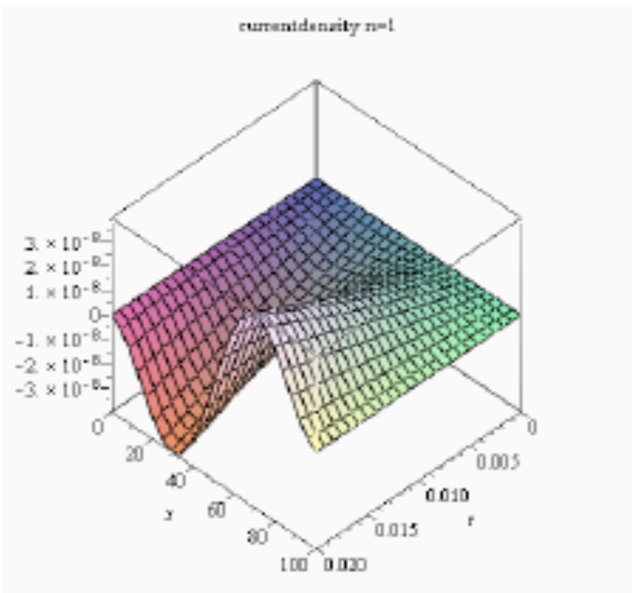


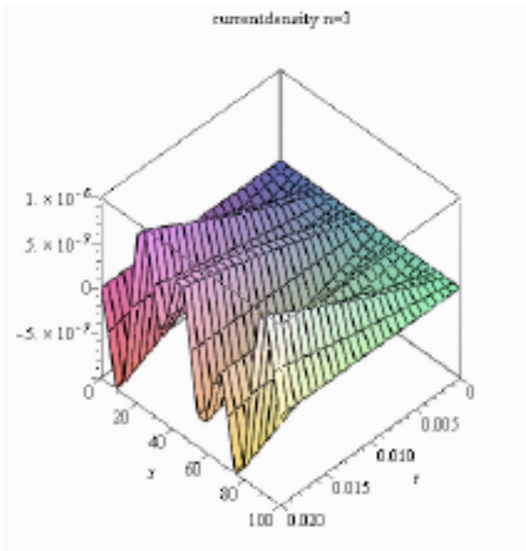






By contrast, below is how a quantum model produces turbulence as the current density of a quantum mechanical system in a one dimensional box of length L. In this model the solution produces turbulence as the quantum number n changes the current density from the left to right and back. The transitions produces a stroboscopic picture of turbulence [3[. It is easy to produce turbulence in a quantum mechanical model, not possible in the classical Navier-Stokes equation as in the first four plots.





References

[1] A. Muriel, An exact solution of the 3D Navier-Stokes equation, Results in Physics, 2-6, 2011.

[2] A. Muriel, Quantum Nature of Turbulence, Nova Science Publishers, New York, 2010

[3] A. Muriel, Quantum justification for the Kolmogorov hypothesis, submitted to JFM.