OBLIQUE LAMINAR-TURBULENT INTERFACES IN TRANSITIONAL SHEAR FLOWS

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Abstract The onset of transition to turbulence in subcritical wall-bounded flows is characterised by large-scale localised structures such as turbulent spots or turbulent stripes. Interestingly, the laminar-turbulent interfaces associated with these structures always display obliqueness with respect to the mean direction of the flow. We will attempt to explain this phenomenon using an assumption of scale separation between large and small scales, and we can show analytically why the corresponding laminar-turbulent interfaces are always oblique with respect to the mean direction of the flow in the case of plane Couette flow. This mechanism can be easily extended to other flows such as Plane Poiseuille flow or Taylor-Couette flow.

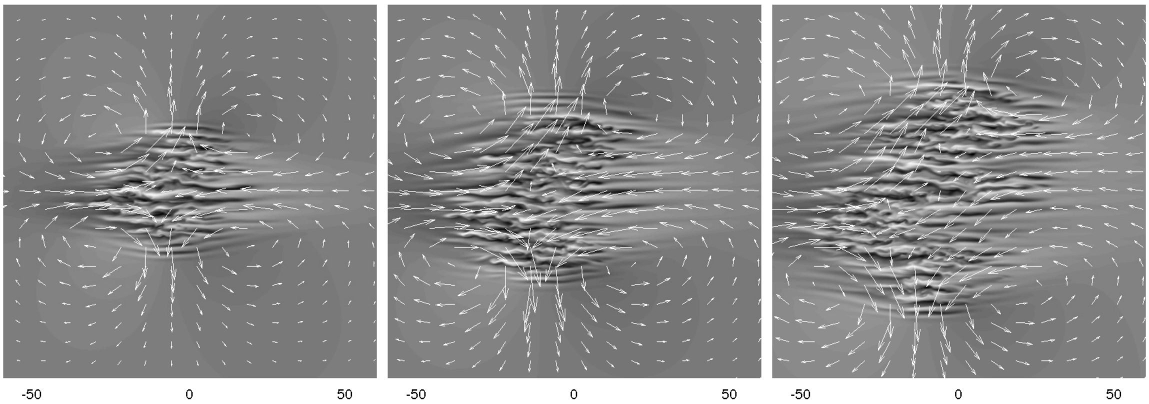

Figure 1. Obliquely growing spot in plane Couette flow at $R = 360$ (grey: streamwise velocity in the midplane) and associated y-integrated large-scale flow $\left(\bar{U}_x, \bar{U}_z\right)(x,z)$ (arrows). From left to right: $t = 200$, 300 and 400. Simulations in a periodic domain with $\Lambda = 500$ and $1536 \times 33 \times 2048$ spectral modes. Only the subdomain $[-60:60] \times [-60:60]$ is displayed here.

An example of formation of large-scale hydrodynamical patterns of co-existing laminar and turbulent flow is found at the onset of transition of plane Couette flow, the flow between two counter-sliding plates. Surprisingly, the disordered phase breaks the symmetries associated with the geometry and adopts an orientation oblique with respect to the mean flow[1]. Especially the origin of the obliqueness of the associated interfaces has long remained mysterious. We assume that the plates move with opposite velocities $\pm U$ in the streamwise direction x. We also define the half-gap h between the plates in the y direction, the spanwise direction z, and the Reynolds number R as Uh/ν, where ν is the kinematic viscosity of the fluid, supposed Newtonian and incompressible. Lengths and velocities are non-dimensionalised by respectively h and U. Direct numerical simulations were carried out with the same pseudo-spectral code as in Ref. [2], in a periodic domain of size $\Lambda \times 2 \times \Lambda$, with $\Lambda = 500$ and with high spectral resolution.

We base our analysis on the existence of two distinct characteristic scales and assume clear spectral separation. The small scales correspond to the coherence of the turbulent fluctuations inside a turbulent patch (streaks) while the large scales correspond to the diffusive tails of the streaks, so that the scale separation grows like $O(R)$. Using the scale separation hypothesis we can separate the flow field $\boldsymbol{u}$ into small scales $\tilde{u} = H\boldsymbol{u}$ and large scales $\boldsymbol{U} = L\boldsymbol{u}$ using adequate plane-isotropic Gaussian low-pass and high-pass filters, respectively H and L. Denoting y-averaging with a bar $\bar{(.)}$, we deduce from the incompressibility of the flow a two-dimensional divergence-free condition in the xz-plane for the large-scale flow:

$$\partial_x \bar{U}_x + \partial_z \bar{U}_z = 0, \tag{1}$$

The growth of a turbulent patch is shown in Fig. 1 for $Re = 360$ along with the corresponding large scale flow $\left(\bar{U}_x, \bar{U}_z\right)$. The streamwise ends of a such a turbulent patch are characterised by so-called overhang regions where locally turbulent flow one one wall faces nearly laminar flow near the other wall. Those regions correspond to a mismatch in the flow rates $\bar{U}_x \neq 0$, whereas $\bar{U}_x = 0$ everywhere else. As a consequence $\partial_x \bar{U}_x \neq 0$ in the overhang regions, hence $\bar{U}_z \neq 0$, and the large-scale flow is locally oblique with respect to the streamwise direction.

In order to understand how the large-scale affects the shape of the laminar-turbulent interface, we use the decomposition introduced earlier and apply successively the filters L and H to the wall-normal momentum equation. The scale separation hypothesis results in a simplified system

$$(\partial_t + \boldsymbol{U} \cdot \nabla) \, U_y = -\partial_y P + R^{-1} \nabla^2 U_y - L \left((\tilde{\boldsymbol{u}} \cdot \nabla) \, \tilde{u}_y \right) \tag{2}$$

$$(\partial_t + \boldsymbol{U} \cdot \nabla) \, \tilde{u}_y = -\partial_y \tilde{p} + R^{-1} \nabla^2 \tilde{u}_y - H \left((\tilde{\boldsymbol{u}} \cdot \nabla) \, \tilde{u}_y \right) \tag{3}$$

The second equation for the small scales indicates that newly nucleated streaks at the tips of the spots will be advected by the large-scale flow, which we know has a non-zero angle with respect to the streamwise direction. As a consequence, the growth of the spots will be distorted by the presence of the large-scale flow and proceed obliquely as well [3].

References

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