DIRECT NOISE COMPUTATION USING A HIGH-ORDER DISCONTINUOUS GALERKIN METHOD

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Abstract The direct numerical simulation of fluid flow with noise generation and propagation needs a very flexible and accurate numerical approach to be efficient. We show that discontinuous Galerkin schemes have a high potential to satisfy these requirements. Especially the discontinuous Galerkin spectral element method is a promising numerical method in this field of application which relies on Cartesian or hexahedral grids. Its main advantages, low dissipation and dispersion as well as high parallel computing efficiency, render the method a natural choice for complex aeroacoustic problems. Numerical examples such as the sound generation of 2-D and 3-D mixing layers demonstrate the ability of the method to reproduce aeroacoustic phenomena efficiently through massively parallel computations. Furthermore, noise generation of wall-bounded flow is presented for a NACA0012 airfoil.

DIRECT NOISE COMPUTATION

In computational aeroacoustics, the hybrid approach, in which flow simulation and acoustic propagation are decoupled, is attractive especially for flow problems at low Mach numbers. Unsteady flow data from mainly incompressible CFD simulations are used as source terms in linear equations such as the linearized Euler equations (LEE) or the acoustic perturbation equations (APE). Analytical methods like the Ffowcs-Williams-Hawkings equation employ surface integrals over a specified source region in order to retrieve the acoustic pressure signal at observer locations. Another class of simulations is based on steady state Reynolds averaged Navier-Stokes (RANS) solutions. Here, time-dependent source terms have to be reconstructed from steady flow and turbulence data using some form of empirical input. However, hybrid and analytical methods suffer from basic problems: There is no feedback from acoustics to the fluid flow and the acoustic sources need to be modeled in some way, adding uncertainty to the simulation.

In an accurate compressible time-dependent Navier-Stokes simulation, the acoustic wave propagation is in principle resolved together with the fluid flow. Within the flow simulation, the acoustic propagation is already included. The major difficulty with this approach lies in the different requirements the fluid flow and the acoustics impose on the numerical scheme. While for the fluid flow simulation, the numerical scheme must capture strong gradients within boundary layers or shock waves, acoustic wave propagation requires a numerical method with low dissipation and dispersion errors. This issue stems from the typically long propagation path from the noise source to the listener, over which the sound has to be transported without significant deterioration of the signal, which necessitates the use of high-order schemes with their associated accuracy properties.

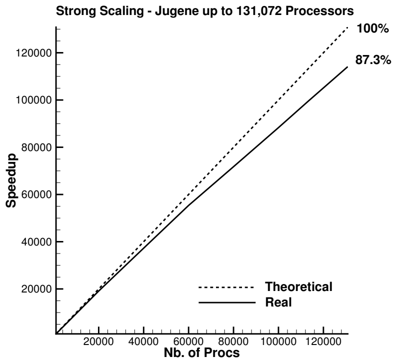
In this paper we show the advantages of discontinuous Galerkin (DG) schemes for direct noise computation (DNC). The inherent flexibility of the DG scheme with respect to computational grid, formal order of accuracy and high computational efficiency on massively parallel computer clusters are the main building blocks to achieve an efficient direct numerical simulation of flow with noise generation in one step. We show results for mixing layer flows and wall bounded flows and analyze and compare our results with those from the literature.

NUMERICAL METHOD

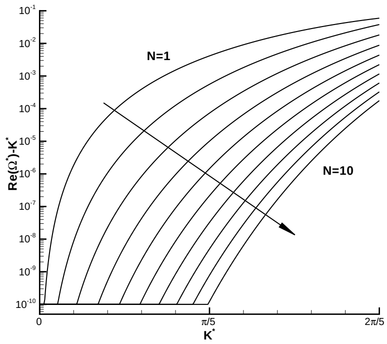
The discontinuous Galerkin method builds upon both finite volume and finite element techniques. Within each grid cell, the solution is represented through local polynomials and is obtained through a variational formulation, while discontinuities of the solution at grid cell interfaces are introduced and treated through numerical flux functions (e.g. Riemann solvers) just as in finite volume methods. While there exist a number of different formulations for different grid cell types like tetrahedra, prisms and pyramids, the most efficient DG formulation yet known is the discontinuous Galerkin spectral element method (DGSEM) proposed by Kopriva [5]. The scheme's restriction to hexahedral grid cells allows a very efficient treatment of the volume integral, as a tensor product of one dimensional Lagrangian basis functions may be employed. Although only one grid cell type is possible with this approach, unstructured grids of curved hexahedra may be used to treat complex geometries.

Since each grid cell only communicates with its direct neighbours through the flux integral, the scheme is inherently parallel and thus well suited for large scale simulations. Strong scaling results with our DGSEM code are shown in Figure 1a. With 131,072 processors (one grid cell per processor), we obtain 87% scaling for a polynomial degree of $N = 7$.

As the dispersion and dissipation properties of the scheme are crucial for the success of a direct noise computation, high-order methods like the DGSEM are a favorable choice. Figure 1b displays the dispersion error of the scheme for increasing polynomial degrees in the well resolved range, clearly demonstrating the superior accuracy of high order methods for wave propagation problems [4].



(a) Strong scaling (fixed problem size).



(b) Dispersion errors for polynomial degrees $N = 1$ to $N = 10$.

Figure 1: Properties of the Gauss DGSEM scheme.

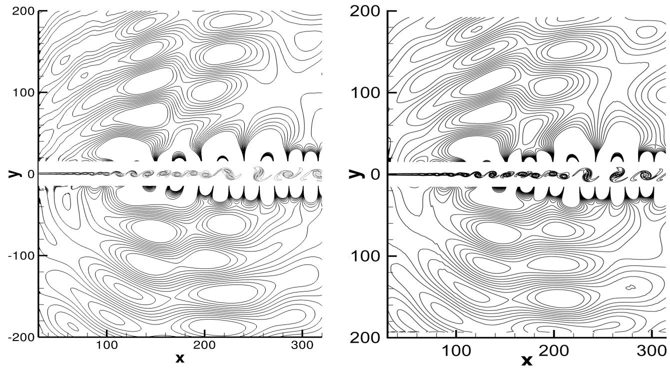


Figure 2: Visualization of the emitted sound with the second subharmonic of the vortex shedding frequency for the 2-D mixing layer using the real part of the Fourier-transformed dilatation $\nabla \cdot \vec{v}$ and comparison to the reference result [1] (right).

COMPUTATIONAL EXAMPLES

The capability of our method to handle aeroacoustic problems is evaluated by direct numerical simulations (DNS) of 2-D and 3-D mixing layers using setups according to Colonius et al. [2] and Babucke [1]. Figure 2 shows the emitted sound of the 2-D mixing layer by means of the Fourier-transformed dilatation field ($\nabla \cdot \vec{v}$) and a comparison with the reference solution (right) by Babucke [1]. We demonstrate that the DGSEM is able to capture the mixing layer's noise generation mechanisms and to properly resolve acoustic wave propagation into the far field. Using this example, the computational efficiency is shown to exceed that of a traditional compact finite difference method of same formal order of accuracy.

In a second step, we will present the application to wall-bounded flows such as trailing edge noise on a NACA0012 airfoil following the example of Desquesnes et al. [3]. The tonal noise source mechanism in this type of flow involves acoustic feedback to the laminar boundary layer, triggering discrete frequency boundary layer instabilities. Clearly, this mechanism can only be reproduced in a simulation by resolving the noise propagation and the flow simultaneously.

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