QFT ANOMALIES IN HYDRODYNAMICS

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Abstract We show that the hydrodynamic limit of a Quantum Field Theory (QFT) with global anomalies has to be modified. This modification leads to new transport phenomena which should be important in heavy-ion collisions and early cosmology. The transport is non-dissipative and related transport coefficients should be universal with respect to QFT coupling constant. We point out future research questions, which include studies of turbulent flows in the presence of QFT anomalies.

Hydrodynamics is an effective field theory which can be used to describe many different physical systems. One of the recent applications of the theory of relativistic hydrodynamics is the study of the evolution of fireball created in heavy-ion collisions the so-called Quark Gluon Plasma. The relativistic hydrodynamic equations have been proposed many years ago [1, 2]; such equations describe the dynamics of an interacting relativistic theory at large distance and time scales. interacting relativistic theory at large distance and time scales. The hydrodynamic variables are the local velocity $u^\mu(x)$ (satisfying $u^2 = -1$), the local temperature $T(x)$ and chemical potential(s) $\mu^a(x)$, where the index a numerates the conserved charges. The hydrodynamic equations govern the time evolution of these variables; they have the form of the conservation laws $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^{a\mu} = 0$, supplemented by the constitutive equations which express $T^{\mu\nu}$ and $j^{a\mu}$ in terms of u^μ, T, and μ^a. These equations are the relativistic generalization of the Navier-Stokes equations.

One feature of relativistic quantum field theory that does not have direct counterpart in nonrelativistic physics is the presence of triangle anomalies [3, 4]. For currents associated with global symmetries, the anomalies do not destroy current conservations, but are reflected in the three-point functions of the currents. When the theory is put in external background gauge fields coupled to the currents, some of the currents will no longer be conserved.

In the simplest case when there is one U(1) current with a U(1)^3 anomaly. We consider only global currents that are not coupled to dynamical gauge fields, and assume the associated symmetries are not spontaneously broken. The constitutive equation for the conserved current j^μ must contains an additional term proportional to the vorticity.

$$j^\mu = nu^\mu - \sigma T(g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left(\frac{\mu}{T}\right) + \xi\omega^\mu, \tag{1}$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho, \tag{2}$$

where n is the charge density, σ is the conductivity, and ξ is the new kinetic coefficient.

Even in a parity-invariant theory, the vorticity-induced current $\xi\omega^\mu$ is allowed by symmetries if, e.g., j^μ is a chiral current. This term contains only one spatial derivative, and its effect is as important as those of viscosity or diffusion. Before very recently, this term has been completely overlooked. In fact, if one follows the standard textbook derivation [2], the new term seems to be disallowed by the existence of an entropy current with manifestly positive divergence, required by the second law of thermodynamics.

In this presentation we will show that this new term is not only allowed, but is required by anomalies. Moreover, the parity-odd kinetic coefficient ξ is completely determined by the anomaly coefficient C, defined through the divergence of the gauge-invariant current, $\partial_\mu j^\mu = -\frac{1}{8}C\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$, and the equation of state,

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right), \tag{3}$$

where ϵ and P are the energy density and pressure. In the case of multiple U(1) conserved currents, the formulas are modified only slightly. Namely, Eq. (3) becomes

$$\xi^a = C^{abc}\mu^b\mu^c - \frac{2}{3}n^a C^{bcd}\frac{\mu^b\mu^c\mu^d}{\epsilon + P}\,, \tag{4}$$

where a, b, c numerate the currents, C^{abc} is symmetric under permutations of indices and is determined from the anomalies, $\partial_\mu j^{a\mu} = -\frac{1}{8}C^{abc}\epsilon^{\mu\nu\alpha\beta}F^b_{\mu\nu}F^c_{\alpha\beta}$. The key ingredient required to derive such transport is the second law of thermodynamics. The entropy current has to be modified to cancel the unwanted terms containing the Levi-Civita symbol which can in principle violate the second law. Therefore the effect is non-dissipative - it does not contribute to the entropy production.

The physical meaning of these new terms can be made explicit by the following example. Consider a volume of rotating quark matter, made of of massless u and d quarks, at baryon chemical potential μ. For a moment let us neglect instanton effects, so the $\mathrm{U}(1)_A$ current $j_5^\mu = \bar{q}\gamma^\mu\gamma^5 q$ is conserved. Due to the triangle anomaly in the three-point correlators of j_5^μ with two baryon currents, Eq. (4) implies that axial current will flow along the axis of rotation: $\langle j_5^\mu \rangle = \frac{3}{\pi^2}\omega^\mu$. This can be thought of as chiral separation [5, 7, 6]: left- and right-handed quarks move with slightly different average momentum, creating an axial current (see Fig. 1). Analogously, in the presence of baryon and isospin chemical potential, the axial

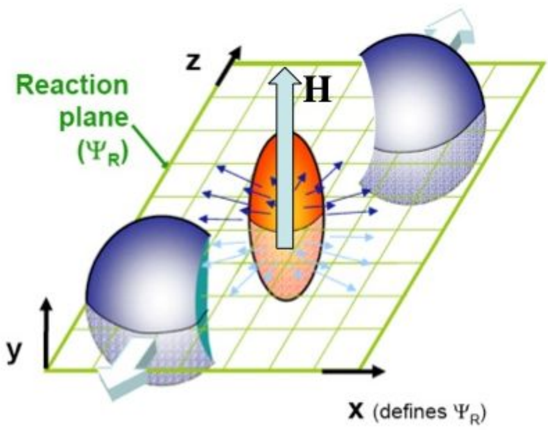

Figure 1. Sketch of the evolution of matter after the collision of two heavy nuclei. The cigar shaped blob of matter is described by hydrodynamics. Due to vorticity and magnetic field particles with different chirality should flow in different directions

isospin current $\bar{q}\gamma^\mu\gamma^5\tau^3 q$ flows along the rotation axis.

The anomaly related transport is non-disspiative. Therefore it is convenient to formulate it in terms of partition functions [8, 9]. Such formulation sheds new light on the quantum origin of the transport and allows for generalizations e.g. include gravitational anomalies [10], construct parity-odd superfluid hydrodynamics [11]. It also creates new research paths, both formal and applied. The formal investigations include a proof that the transport coefficients do not change under Renormalization Group transformations, analysis of turbulent flows in the presence of anomalies, connection with topology. The applied research line include the analysis of the "anomalous" transport in Weyl semi-metals and superfluids and the connection between Berry phases and anomalies, moreover experimental verification of such effects should be possible both in condensed matter systems and heavy-ion collisions.

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