RECEPTIVITY OF FLAT-PLATE BOUNDARY LAYER TO NON-LINEARLY DEVELOPING FREE-STREAM TURBULENCE

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Abstract Linear receptivity of flat-plate boundary layer to non-linearly developing free-stream turbulence is considered. In contrast to general approach free-stream vortical disturbances moving with phase speed deviating from flow velocity are considered. This makes possible to describe correctly evolution of velocity pulsations in boundary layer and its frequency spectrum.

INTRODUCTION

The effect of free-stream turbulence (FST) on laminar turbulent transition in a boundary layer has become of great interest during the last decade. General consensus is that boundary layer disturbances in this conditions grow proportionally to Reynolds number based on the boundary layer thickness. It means that transition Reynolds number should be determined by the turbulence intensity only. However the discrepancy in published observations of transition is substantial (see [1]). Linear theory of boundary layer receptivity developed by Leib, Wundrow & Goldstein [3] states that r.m.s. pulsations in boundary layer is described by the universal law $u'/(Tu\sqrt{R_L}) = F(\sqrt{R_x}/R_L)$ where R_L - is Reynolds number based on integral scale of turbulence *L*. Results of experiments [1,2] scaled in this way are presented in Fig.1, *a*. It shows that linear theory [3] strongly underestimates magnitude of pulsations. Discrepancy between the predictions of linear flat-plate boundary layer receptivity theory and experiment may be caused by non-linearity. There are two types of non-linear effects in the FST-induced transition: non-linear evolution of vortical disturbances in outer flow and non-linear development of streaky structures in the boundary layer. Here we shall account the first type of non-linear effect and describe the linear development of disturbances in boundary layer initiated by non-linear turbulence in the outer flow.

MODEL OF STREAMWISE VORTICITY OF FST

Let's consider the interaction of grid turbulence with the boundary layer at infinitely thin plate. The oncoming flow has mean velocity u_∞ and r.m.s. pulsations $u' = Tu u_\infty$. We introduce non-dimensional variables using free-stream velocity and viscose length $l = \nu/u_\infty$ as scales. In these variables all coordinates are equal to the corresponding Reynolds numbers. In accordance to general agreement flat plate boundary layer is most receptive to streamwise vorticity of FST. It will be presented as a superposition of vortical modes $\omega_{xe} = a(x)\exp\{i(\alpha(x-t)+\beta y+\gamma z-\tilde{\omega}t\}$ which are periodic in space and time. Here α,β,γ - are streamwise spanwise and vertical wavenumbers, $\tilde{\omega}$- is frequency. Further it is assumed that spanwise and vertical periods of vortical modes are large, so cross-flow wavenumbers are small and will be considered as small parameters $\beta \sim \gamma << 1$. Low-frequency disturbances with $\alpha \sim \tilde{\omega} \sim \beta^2$ will be considered further because of such perturbations exhibit maximal algebraic growth in the boundary layer.

In classical linear receptivity theory the interaction between vortical modes is neglected and they correspond to solutions of linearized Navier-Stokes equations. Amplitude of such modes decays exponentially and they are convected with free-stream velocity, so $\tilde{\omega} = \alpha$. In real turbulence disturbances decay more slowly and their phase speed deviates from the free-stream velocity. For this reason we shall consider vortical modes with arbitrary dependence of amplitude from *x* and detuned frequency $\tilde{\omega} = \alpha + \omega$. Such modes can not exist without the interaction with other part of spectrum of FST. The action of other disturbances to the mode will be replaced by the external force ***F.***

Free stream vorticity of FST is characterized by its wavenumber-frequency spectrum $G_f(\mathbf{k},\omega,x)$, so amplitude of each elementary vortical mode is determined as $a(k,\omega,x) = \sqrt{G_f(k,\omega,x)}$. Further we shall assume that spectrum of streamwise vorticity of FST can be presented as a product $G_f(\mathbf{k},\omega,x) = G(\mathbf{k},x)S(\omega,x)$ where $G(\mathbf{k},x)$ and $S(\omega,x)$ are wave-number and frequency spectra of ω_x. Wave-number spectrum of streamwise vorticity is related to 3d energy spectrum of FST *E(k)* as

$$G(\mathbf{k},x) = \frac{1}{4\pi}E(k,x); \quad E(k) = Tu^2 LF(k_1); \quad F(k_1) = \frac{15}{12\pi}\frac{k_1^{\,4}}{(1+k_1^{\,2})^{17/6}}; \quad k_1 = kL$$

Here $F(k_1)$ is normalized 3d energy spectrum which is approximated by Karman's spectrum. Frequency spectrum of FST can be related with characteristic correlation time of velocity pulsations τ found in [4]

$$S(\omega) = \frac{\tau}{\sqrt{\pi}}\exp\left\{-\frac{1}{2}(\omega\tau)^2\right\}; \quad \tau = \lambda\frac{L}{Tu}B(k_1)$$

RECEPTIVITY OF BOUNDARY LAYER TO FST

Disturbances produced by vortical mode in the boundary layer $\mathbf{v}(x,y,z,t)$ are governed by Navier-Stokes equations linearized around the basic flow in the boundary layer V_b. Due to large streamwise period of perturbations the streamwise pressure gradient can to be neglected and these equations take form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{V}_b, \nabla)\mathbf{v} + (\mathbf{v}, \nabla)\mathbf{V}_b = \nabla_\perp p + \Delta \mathbf{v} + \mathbf{F}; \quad (\nabla, \mathbf{v}) = 0$$

This set of equations is of parabolic type, so initial conditions for $x=0$ and boundary conditions at the plate and in the outer flow are necessary. No-slip conditions are set at the plate, initial and outer flow conditions correspond to cross-flow velocity induced by vortical mode in the free stream. Based on solution for single vortical mode $u(k,\omega,x)$, boundary layer velocity pulsations from oncoming turbulence can be expressed as an integral

$$< u'^2 >= \int\iint_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} G_f(k,\omega,x) |u(k,\omega,x)|^2 d\omega \right\} \mathbf{dk}$$

Further considerations shows that velocity pulsations in boundary layer can be presented in folloiwng form

$$\frac{\sqrt{< u'^2 >}}{Tu_0\sqrt{L_0}} = \Phi\left(\frac{x}{L_0{}^2}, R_t\right); \quad R_t = Tu_0 L_0 \tag{1}$$

where Tu_0, L_0 are turbulence intensity and scale at the leading edge and R_t is turbulent Reynolds number. Constant $\lambda = 0.2$ was found from the best fit of this solution with experimental data.

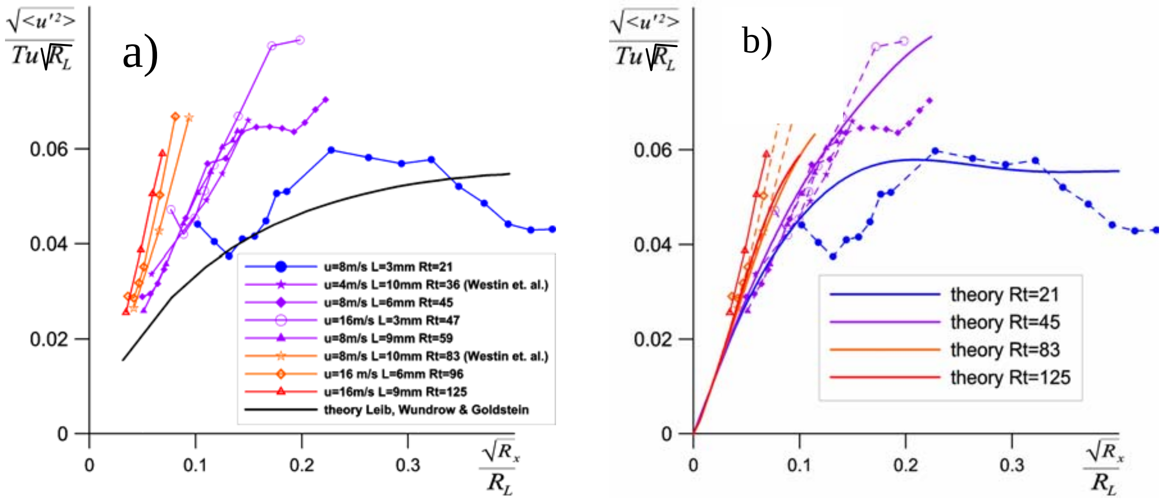

Figure 1. Pulsations in the boundary layer normalized in accordance with (1). Experimental data denoted by symbols. Thick black line – linear theory [3], color solid lines – results of present non-linear receptivity theory.

Amplification curves of pulsations in the boundary layer computed for R_t=21, 45, 83, and 125 are shown in Fig. 1,b. Coincidence of developed non-linear receptivity theory with experiment is rather good in comparison with linear receptivity theory [3]. Main advantage of present theory is qualitative description of the enhancement amplification coefficient with the growth of turbulent Reynolds number. However, theory underestimates this trend.

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