TURBULENT MAGNETIC PRANDTL NUMBER AND SPATIAL PARITY VIOLATION

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Abstract Using the field theoretic renormalization group technique in the two-loop approximation the influence of helicity (spatial parity violation) on the turbulent magnetic Prandtl number is studied in the model of the kinematic magnetohydrodynamic turbulence, where the magnetic field behaves as a passive vector quantity advected by the helical turbulent environment given by the stochastic Navier-Stokes equation. It is shown that the presence of helicity decreases the value of the turbulent magnetic Prandtl number and that the two-loop helical contribution to the turbulent magnetic Prandtl number is up to 4.2% of its non-helical value.

INTRODUCTION

One of the characteristics of diffusion processes of the magnetic field in a conductive medium is the dimensionless ratio of the coefficient of the kinematic viscosity to the coefficient of the magnetic diffusivity (resistivity), namely, the so-called magnetic Prandtl number. When the conductive medium is in the state of fully developed turbulence, i.e., in the state of magnetohydrodynamic (MHD) turbulence, then the diffusion processes are rapidly accelerated and are described by an effective value of the coefficient of magnetic diffusivity, namely, by the so-called turbulent magnetic diffusivity (resistivity). The ratio of the turbulent viscosity to the turbulent magnetic diffusivity is usually called the turbulent magnetic Prandtl number $Pr_{m,t}$ [1].

Recently the field theoretic renormalization group (RG) technique was used for the calculation of the two-loop value of the turbulent magnetic Prandtl number in the framework of the kinematic MHD turbulence with fully symmetric isotropic turbulent environment given by the stochastic Navier-Stokes equation [2]. In Ref. [2], it was shown that the two-loop correction to the turbulent magnetic Prandtl number is surprisingly very small (it is less than 2% of its one-loop value). Although, it seems that the turbulent magnetic Prandtl number is stable under the perturbation expansion, nevertheless a few interesting questions are still open. For example, it is the question of the influence of various symmetry breaking on the turbulent processes deep inside of the inertial interval. The importance of this question is dictated by the well-known experimental fact that the homogenization processes of the initial and boundary conditions (energy pumping at large scales) which lead to the developed turbulence are considerably slower than it was assumed earlier (see, e.g., Ref. [3] and references cited therein).

In this respect, we shall investigate the advection processes of a weak magnetic field by the turbulent environment given by the stochastic Navier-Stokes equation driven by the helical random force, i.e., by the random force with the spatial parity violation, in the framework of the kinematic MHD turbulence. Our aim is to find the explicit dependence of the turbulent magnetic Prandtl number on the parameter which controls the presence of helicity in the turbulent system. We shall show that the presence of helicity can have nontrivial impact on the value of the turbulent magnetic Prandtl number and, consequently, on the properties of diffusion processes of a vector (magnetic) admixture in fully developed turbulent environments.

THE MODEL

The advection of passive solenoidal magnetic field $\mathbf{b} \equiv \mathbf{b}(x)$ ($x \equiv (t, \mathbf{x})$) in a helical incompressible turbulent environment in the framework of the kinematic MHD turbulence is given by the following system of stochastic equations:

$$\partial_t \mathbf{b} = \nu_0 u_0 \triangle \mathbf{b} - (\mathbf{v}\cdot\partial)\mathbf{b} + (\mathbf{b}\cdot\partial)\mathbf{v} + \mathbf{f}^{\mathbf{b}}, \quad (1)$$

$$\partial_t \mathbf{v} = \nu_0 \triangle \mathbf{v} - (\mathbf{v}\cdot\partial)\mathbf{v} - \partial P + \mathbf{f}^{\mathbf{v}}, \quad (2)$$

where the standard notation is used: $\partial_t \equiv \partial/\partial t$, $\partial_i \equiv \partial/\partial x_i$, $\triangle \equiv \partial^2$ is the Laplace operator, ν_0 is the viscosity coefficient, u_0 is the reciprocal magnetic Prandtl number, $\mathbf{v} \equiv \mathbf{v}(x)$ is the incompressible velocity field, and $P \equiv P(x)$ is the pressure. The explicit form of the magnetic random noise $\mathbf{f}^{\mathbf{b}} = \mathbf{f}^{\mathbf{b}}(x)$ is not important in what follows. On the other hand, the explicit form of the transverse random force per unit mass $\mathbf{f}^{\mathbf{v}}$ is essential and we assume that it obeys a Gaussian distribution with zero mean and correlator (see Ref.[2] for details)

$$D^v_{ij}(x;x') = \langle f^{\mathbf{v}}_i(x) f^{\mathbf{v}}_j(x')\rangle = \delta(t-t') g_0 \nu_0^3 \int \frac{d^d\mathbf{k}}{(2\pi)^d} R_{ij}(\mathbf{k}) k^{4-d-2\varepsilon} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}, \quad (3)$$

where d is the dimension of the $\mathbf{x}$ space, the most realistic value of exponent $0 < \varepsilon \leq 2$ is $\varepsilon = 2$, g_0 is the coupling constant (a formal small parameter of the ordinary perturbation theory), and $R_{ij}(\mathbf{k})$ is a transverse projector. In our isotropic helical case, the transverse projector $R_{ij}(\mathbf{k})$ consists of the non-helical standard transverse projector $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$ and $H_{ij}(\mathbf{k}) = i\rho\,\varepsilon_{ijl}k_l/k$, which represents the presence of helicity in the flow. Thus, the transverse projector $R_{ij}(\mathbf{k})$ in Eq. (3) has the form

$$R_{ij}(\mathbf{k}) = P_{ij}(\mathbf{k}) + H_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2 + i\rho\,\varepsilon_{ijl}k_l/k, \tag{4}$$

Here, ε_{ijl} is the Levi-Civita's completely antisymmetric tensor of rank 3 and the real parameter of helicity ρ characterizes the amount of helicity. Due to the requirement of positive definiteness of the correlation function the absolute value of ρ must be in the interval $|\rho| \in [0, 1]$. Physically, the nonzero helical part expresses the existence of nonzero correlations $\langle \mathbf{v} \cdot \mathrm{rot}\, \mathbf{v} \rangle$ in the system.

HELICITY AND THE TURBULENT MAGNETIC PRANDTL NUMBER

Using the field theoretic formulation of the model of the kinematic MHD turbulence the scheme independent two-loop RG approximation formula for the inverse turbulent (effective) magnetic Prandtl number u_{eff} can be derived in the following form (see Ref.[2] for more details)

$$u_{eff} \;=\; u_*^{(1)}\left(1+\varepsilon\left\{\frac{1+u_*^{(1)}}{1+2u_*^{(1)}}\left[\lambda - \frac{128(d+2)^2}{3(d-1)^2}B(u_*^{(1)})\right] + \frac{(2\pi)^d}{S_d}\frac{8(d+2)}{3(d-1)}(a_v - a_b)\right\}\right). \tag{5}$$

Here, $u_*^{(1)}$ is the one-loop fixed point value of the parameter u (it is also the one-loop value for the inverse turbulent magnetic Prandtl number). For $d = 3$ one has $u_*^{(1)} = 1.39297$. The quantities λ and $B(u_*^{(1)})$ are given by the explicit calculations of the corresponding two-loop Feynman diagrams (see Ref.[2] for details). The quantity λ does not depend on helicity parameter ρ and for $d = 3$ one obtains $\lambda = -1.0994$. On the other hand, it can be shown that the explicit dependence of the quantity $B(u_*^{(1)})$ for $d = 3$ is given as follows: $B(u_*^{(1)}, \rho) = -4.4320\times10^{-3} - 0.1326\times10^{-3}\rho^2$. Further, the quantities a_v, a_b in Eq. (5) are given by the corresponding expansions to the leading order in ε of the scaling functions of the response functions $\langle vv' \rangle$ and $\langle bb' \rangle$ for the velocity field and the magnetic field, respectively (see Refs. [2] for details). They are independent of helicity and their values for $d = 3$ are $a_v = -0.047718/(2\pi^2)$ and $a_b = -0.04139/(2\pi^2)$. Now, using all these facts together with the assumption $d = 3$ in Eq. (5) one comes to the final two-loop explicit dependence on the helicity of the turbulent magnetic Prandtl number in the framework of the kinematic MHD turbulence (for the physical value $\varepsilon = 2$)

$$Pr_{m,t} = u_{eff}^{-1} = \frac{1}{1.42046 + 0.06229\rho^2}. \tag{6}$$

In the limit $\rho \to 0$ one comes to the nonhelical value $Pr_{m,t} = 0.7040$. On the other hand, in fully helical case, i.e., when $|\rho| = 1$, one has $Pr_{m,t} = 0.6744$. Besides, looking at Eq. (6), one can conclude that the turbulent magnetic Prandtl number decreases in helical turbulent environment, i.e., when the absolute value of the parameter ρ increases. At the same time, the two-loop helical contributions to the turbulent magnetic Prandtl number are at most 4.2% (in the case with the maximal helicity) of its nonhelical value.

CONCLUSION

We have used the field theoretic RG approach in the two-loop approximation to analyze the influence of the presence of helicity (spatial parity violation) in fully developed turbulent flows on the diffusion processes of a passively advected magnetic field in the framework of the kinematic MHD turbulence. The turbulent magnetic Prandtl number is found as function of the helicity parameter. It is shown that the helicity decreases the value of the turbulent magnetic Prandtl number. Although, the two-loop helical corrections to the turbulent magnetic Prandtl number are rather small (up to 4.2% of its nonhelical value), nevertheless the obtained results demonstrate the fact that deviations from fully symmetric turbulent systems can have nontrivial impact on the universal characteristics of the processes in turbulent environments.

References

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