TURBULENT FLOW OF SUPERFLUID ^{4}HE THROUGH SQUARE DUCTS

Simone Babuin[1], Emil Varga[2] & Ladislav Skrbek[2]
[1]*Institute of Physics ASCR, Na Slovance 2, 182 21 Prague, Czech Republic*
[2]*Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 121 16 Prague, Czech Republic*

Abstract We report an experimental study of turbulent pipe flow with superfluid ^{4}He as a working fluid, through smooth square ducts of width 7 and 10 mm, and 115 mm in length. The helium temperature is in the range 1.35 K $< T <$ 2.16 K, which corresponds to varying the superfluid fraction respectively from 94% to 9%, that is, from a quasi-ideal to a quasi-viscous fluid. The flow is generated by a low temperature bellows capable of producing well controlled steady flow velocities up to 1 m/s in the duct. We have studied the case of a straight unobstructed duct, and measurements in the same duct with the addition of a grid are under progress. We have studied the dependence of the total length of quantized vortex lines per unit volume (a quantification of total vorticity per unit quantum circulation) as a function of mean velocity and also its decay as a function of time when the turbulence drive is suddenly stopped. Instructive comparisons with a previous experiment where the viscous component of helium was prevented from flowing through the channel are made, and interpretation of measurements based on existing and new models of quantum turbulence are suggested.

INTRODUCTION

Superfluidity is a fundamental phase of matter which arises when a large fraction of the system's particles condense into a single quantum state, coherent over a macroscopic length scale [9]. This phase transition occurs to electrons in superconductors, some cold atomic gasses, liquid ^{4}He and ^{3}He, and within cores of neutron stars. In this project we are interested in the hydrodynamic properties of superfluid ^{4}He, and specifically its turbulent state.

Helium has its boiling point at $T = 4.2$ K, at the saturated vapour pressure. In the normal liquid phase it behaves in all respects as an ordinary Newtonian fluid of very low kinematic viscosity of order 10^{-4} cm^2/s. Below the superfluid phase transition, at $T = 2.17$ K, liquid helium is phenomenologically treated as a mixture of two interpenetrating fluids [8]: a normal component which contributes to all total entropy and viscosity, and a superfluid component which is an ideal liquid with zero viscosity and zero entropy. The superfluid and normal component densities add up to the total helium density, $\rho = \rho_s + \rho_n$, and the superfluid fraction ρ_s/ρ varies from 0 at the transition to 1 in the $T \to 0$ limit. At sufficiently low velocities of order cm/s the superfluid and normal components have independent velocity fields, $\mathbf{v}_s$ and $\mathbf{v}_n$. At higher velocities, when turbulence sets in, the two velocity fields become coupled, the extent of coupling increasing with increasing temperature.

Turbulence in helium, as in classical fluids, always involves some form of rotational motion. For the superfluid component, which for quantum mechanical reasons must have an irrotational velocity field, this can only be realised by the spontaneous nucleation of thin vortex lines [8]. In the zero temperature limit, when all helium is superfluid, the fluid's total vorticity is concentrated within the thin vortex cores of atomic size, around which the superfluid velocity field remains irrotational, with fixed circulation of 10^{-3} cm^2/s. The spatial distribution of the lines depends on the system geometry and on how the turbulence is generated, and in the most general case it forms a complex tangle. Vortex lines can interact with each other via reconnections which conserve circulation, and they can bundle-up to deliver macroscopic fluid motion to scales larger than the typical separation between lines [3]. At finite temperature, the normal component is present, which may or may not be turbulent. The normal component velocity field is affected by the presence of vortex lines because these exert on it a (mainly) dissipative force [8].

EXPERIMENTAL RESULTS AND DISCUSSION

We are studying turbulent pipe flows with superfluid ^{4}He through smooth square ducts of width 7 and 10 mm, and length 115 mm. The helium temperature is in the range 1.35 K $< T <$ 2.16 K, which means that we can vary the superfluid fraction respectively from 94% to 9%, that is, from a quasi-ideal to a quasi-viscous fluid. We generate the flow with a low temperature bellows capable of producing well controlled steady flow velocities, v, up to nearly 1 m/s in the duct. We have studied the case of a straight unobstructed duct, and the study of the same duct with the addition of a grid is under progress. The flow is conditioned at the entrance of the duct by a tight packing of straight tubes with 1 mm diameter at 6 mm length. These experiments can loosely be characterized as extending the investigations made by Van Sciver's team in the 1990s [5]. Turbulent superfluid pipe flow has also been intensely studied in Grenoble by Roche's group and collaborators, focussing though on the determination of vortex line density fluctuation and velocity fluctuation spectra measured with miniature sensors [6, 7].

In the present study, the key quantity measured is the total length of vortex lines per unit volume, L. This is deduced from measuring the attenuation of a temperature wave propagating perpendicular to the mean flow direction, at mid channel length [1]. In superfluid helium temperature can travel as a wave (called second-sound) because of the independent motion of normal and superfluid components, whose relative density can oscillate whilst the total helium density remains nearly constant [9]. Thus, second-sound resonances are excited across the channel by means of special transducers. The presence of vortex lines introduces extra dissipation which appears in the form of attenuated resonant curves, from which L can be inferred. Assumptions on the spacial distribution of the lines are necessary, and introduce an uncertainty on L of about 30%. In classical fluid-dynamics terms, the meaning of L is that of total mean vorticity per unit circulation.

The steady-state turbulence problem is typically studied by examining the features of the $L(v)$ function, which, in general, for a given flow, may also be a function of temperature. A previously published study was dedicated to pure superflow, i.e. the net flow of the superfluid component only, where the flow of the normal component is prevented by plugging the channel ends with superleak filters [1]. The normal and superfluid components are thus forced by construction to undergo relative motion within the channel. We have found $L(v) = K_1(v - v_c)^2$, where $K_1(T)$ is a well-defined temperature dependent prefactor in agreement with several previous pure superflow and thermal counterflow (thermally excited relative motion of the two components) studies, and v_c is the turbulence onset velocity.

In the present co-flow study, on the other hand, the motion of the two components is not forcefully separated, both are subjected to the same pressure gradient provided by the bellows, and their strength of coupling depends on temperature. We have observed significant differences from the pure superflow case. In co-flow $L(v) = K_2(v - v_c)^{3/2}$ and K_2 is strictly temperature independent within the experimental resolution, even when the superfluid density fraction is varied by a factor of 10.

The different power law exponents, 2 and 3/2 – respectively for pure superflow and co-flow, and the different response to temperature change, are currently being explained using existing and new models of quantum turbulence.

We also study the temporal decay of vortex line density, L, when the flow drive is suddenly stopped. This is equivalent to the important classical fluid dynamic problem of the decay of vorticity in relation to the steady-state turbulent energy spectrum. For the pure superflow case we observed different temporal decay behaviour which strongly depends on the initial steady state line density [2]. For a sufficiently dilute tangle we observed $L \propto t^{-1}$, whilst for denser tangles we observed $L \propto t^{-3/2}$, corresponding respectively to an original tangle without and with large scale correlations of the vortex lines [4]. The analysis of the decay of vortex line density from the pipe co-flow experiment is currently underway.

The study of fully developed superfluid turbulence in a pipe – a setting which is very well characterized and studied in classical fluid dynamics – ought to prove very useful to learn further about similarities and differences between quantum and classical turbulence.

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