The Langevin-like model of an anomalous transport

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Abstract A model of the turbulent transport based on the fractional Langevin-like equations was proposed by as in this paper. Parameters depending on a specific type of turbulence are an order of the temporal and spartial derivative and the type of a nonlinearity. A power-law dependence of the particle moving in a time t was proved, and the critical exponent was found.

The solution of fractional dynamical equations

One of attempts to explain the anomalous transport, which began widely developed since 2000-s, is using of the fractional derivatives [1]. This is due to the fact that even in the linear fractional diffusion equation the anomalous scaling is observed. A more realistic formulation of the turbulent problem is one with non-linear equations corresponding to an interaction of a some number of harmonics. This is confirmed by a lot of experimental data [2].

In our approach [3] we substituted in the standard Langevin equation the Caputo operator of order alpha instead of the temporal derivative and the spatial Laplacian of order sigma instead of the standard. Furthermore, the nonlinear interaction of m harmonics was added. The correlation function of a noise was assumed Gaussian. The main object of the study is the dynamical critical exponent z.

It is naturally to require that the exponent z is a continuous function of the parameters alpha and sigma. We have proved [3] that this continuity exists only if m = 3. The problem of finding of the critical exponents was reduced by us to a scalar field theory [1]. We had used an auxiliary small parameter - epsilon and obtained the following results: in the zero-order perturbation theory in epsilon the critical exponent coincides with the well-known value sigma \ alpha, the next (second) order requires the calculation of integrals of the products Fox's H-functions. These integrals were calculated by us using software in the case of alpha is 1 \ 2 and sigma is 2. With these values, we have z = 4 +0.1555 * (3) ^ 2.

In comparison with the integer case the dynamical critical exponent and the role of correction to it increase. In the case of alpha = 1 this correction is 0.66 percent, in our case, it is 26 percent.

References

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