PHYSICAL IMPLICATION OF TWO PROBLEMS IN e-N METHOD FOR TRANSITION PREDICTION

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Abstract The e^N method is widely used in engineering for transition prediction. When it is applied to three-dimensional boundary layers, one has to choose the direction along which the growth rate of the disturbance is to be integrated. The direction determined on the base of saddle point method is mathematically most reasonable. However, unlike in the case of water wave, in problems of hydrodynamic stability, its physical meaning is not so obvious, as the frequency and wave number may be complex. And even more, on some occasions, one may not be able to find the direction on the base of continuously varying the frequency and wave number. In this paper, these problems are clarified by investigating the evolution of a wave packet through DNS. Suggestions for how to do transition prediction are also provided for the case of discontinuity of direction set by saddle point method.

INTRODUCTION

Boundary-layer transition prediction is an important problem both from theoretical and practical points of view. Up to date, the only method which has a sound physical basis and also is widely used in engineering applications is the e^N method. It is based on linear stability theory from which an eigenvalue problem is formulated. For a spatial problem, the frequency is real, and the wave number can be complex. In the e^N method, one has to follow a wave with given frequency, calculate its amplification rate, and then integrate the amplification rate until its amplitude is amplified up to the e^N times of the initial amplitude. The integration is carried on the path of the wave. For a fixed frequency, there can be countless eigenvalue solutions with different wave numbers. How to choose the one that determines the final transition location among all of waves having the same frequency is a key issue in using the e-N method. Cebeci and Stewartson[1-2] used the saddle point method of complex function theory to investigate the evolution of a wave packet with complex wave number. According to them, one should choose such a wave that its wave number satisfies the condition $(\partial\alpha/\partial\beta)_i = 0$, and the propagation direction of the wave is given by $\phi = \tan^{-1}\{-(\partial\alpha/\partial\beta)_r\} = 0$. The strategy proposed by Cebeci and Stewartson has the soundest basis from mathematical point of view. However, its physical meaning is not clear enough. Even more, on some occasions, one may not be able to find such a direction on the base of continuously varying wave number. In this paper, we will clarify these problems by investigating the propagation of a wave packet through direct numerical simulation (DNS), and also study a problem in which wave satisfying the condition set by Cebeci and Stewartson may not be able to find at a certain point under the condition of continuously varying the wave number. Suggestions will be given for how to deal with such a problem in transition prediction.

NUMERICAL METHODS AND RESULTS

The propagation of a wave packet in boundary layers on a flat plate is considered. Two cases are computed as shown in table 1, where the Reynolds number is defined by the displacement thickness at inlet of the computational domain, which is far downstream from the leading edge, and the oncoming flow quantities such as velocity, temperature, density and viscosity coefficient. The wall is adiabatic and the temperature of the oncoming flow is 79K.

The governing equations are compressible full Navier-Stokes equations. A 5^{th} order upwind scheme is used for the split nonlinear term, and a 6^{th} order central scheme is used for viscous term. The 3^{rd} order Runge-Kutta scheme is used for the time advancing.

After a steady 2-D base flow is obtained, disturbance in the shape of a wave packet in span-wise direction is introduced at the inlet, which is given by

$$q'(x,y,z,t) = G(z)\hat{q}(y)e^{i(\alpha x+\beta z-\omega t)} + c.c.,\quad G(z) = A_0 e^{-a(z-z_L/2)^2}$$

where $\hat{q}(y)$ is the eigen-function of O-S equation with given frequency ω and span-wise wave number β, A_0 is the initial amplitude, a very small number as 1.5×10^{-8}. All the parameters are shown in table 1. For case II, two wave packets having different spanwise wave numbers are introduced simultaneously.

Table 1. Parameters in DNS computation

Case	Mach number	Reynolds number	$x_L \times y_L \times z_L$	Mesh	a	ω	β
Case I	3	5×10^4	400×50×390	400×200×600	-0.008	0.14	0.48
Case II	6	10^4	200×25×200	800×150×380	-0.003	0.84	0.797,0

Case I

Figure 1 shows the distribution of initial fluctuating stream-wise velocity along the spanwise direction at y_{max} where $\hat{q}(y_{max})$ attains the maximum value in normal direction. The amplitude of wave packet as shown in figure 2 is obtained as follows, first, for a given x, search the maximum stream-wise fluctuating velocity within a time period for each mesh point, and then find the maximum among all the points, it is the value shown in figure 2. The curve shown in figure 3 labeled DNS is the loci of points having the maximum stream-wise fluctuating velocity for each x. The e-N integral based on the saddle point method is also used to compute the amplitude amplification of the T-S wave and the wave orientation (labeled by SPM). The comparisons with those from DNS are shown in figure 2 and 3 respectively. The fairly good agreements show that the e-N method based on the saddle point method reflects the evolution, both in direction and magnitude, of the peak of a wave packet.

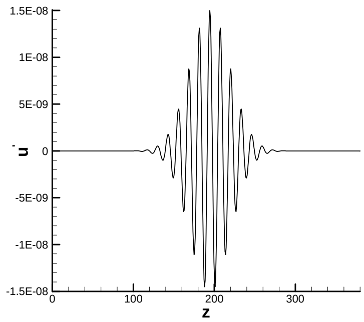

Figure 1 Distribution of flunctuating velocity along z

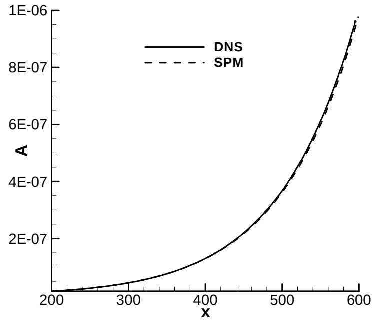

Figure 2 Comparison of amplitude amplification

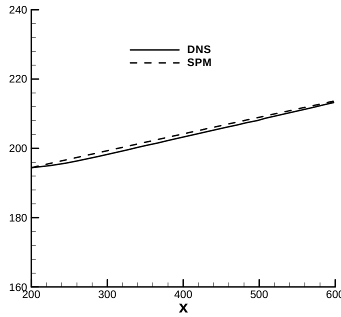

Figure 3 Comparison of wave orientation

Case II

Linear stability analysis is performed for profiles of the base-flow at different stream-wise locations. Assuming β being real, then the variation curve of the stream-wise growth rate α_i against span-wise wave number β is shown in figure 4. For real β, Cebeci and Stewartson's condition corresponds to the extremum of the curve. At x=100, there are three extremum points, namely point A, B and C, among which we can take only two of them into account, namely, A, B or C. A corresponds to a 2-D wave, and B corresponds to a 3-D wave. At x=100, B is more unstable than A, hence should be the first candidate in e^N method. However, beginning from x=148, B is no longer an extremum point, so according to Cebeci and Stewartson, B should be dropped out from the consideration. On the other hand, at first, A is least unstable wave compared with its neighboring wave, but starting from somewhere between x=131-135, it becomes the most unstable one. If we look at the wave packets A and B in spectral space shown in figure 5, which is obtained by DNS, it is clear that at first, the amplitude of wave packet B grows more rapidly than the amplitude of A, but later, it almost ceases to grow, while the amplitude of wave packet A keeps growing and eventually becomes far bigger than that of B. Hence, in transition prediction, wave A would be the one likely to trigger transition.

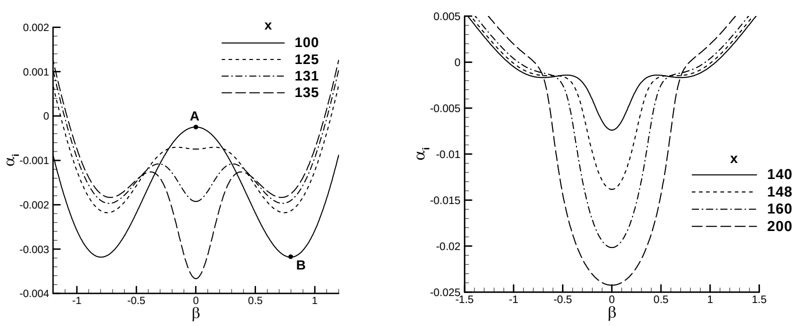

Figure 4 Linear stability analysis of baseflow: (a) x=100~135, (b) x=140~200

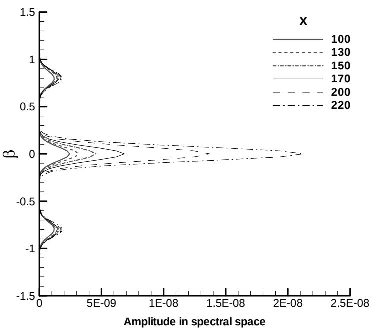

Figure 5 Amplitude of disturbances in spectral space

CONCLUSIONS

1. For the case of Mach number 3, the results from DNS and the saddle-point e-N method agree with each other very well, implying that this method reflects the evolution of peaks in form of a wave packet.
2. For the case of Mach number 6, there is a possibility that condition of Cebeci and Stewartson may ceased to be satisfied at a certain point. In this case, another wave number β satisfying the condition of Cebeci and Stewartson should also be tested, though initially, it may be less unstable than the other one. Physically, it may be due to the fact that when the Mach number is over 4, there may be a competition between 3-D first mode unstable wave and 2-D second mode unstable wave, and the former is oblique wave, while the latter is 2-D wave.

References

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[2] T. Cebeci, J. P. Shao, H. H. Chen, et al. The Preferred Approach for Calculating Transition by Stability Theory. *In: Proceeding of International Conference on Boundary and Interior Layers*, ONERA, Toulouse France, 2004.