DYNAMICAL INTERPRETATION OF MULTIFRACTAL VELOCITY STATISTICS

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Abstract Multiscaling statistics of velocity increments in hydrodynamical turbulence, i.e. nonlinear dependence of their moments, is derived using a simple one-dimensional stochastic equation modeling the flow inside a vortex filament. The main idea is that random oscillations of large-scale velocity result in systematical stretching of a filament. Stretching produces power-law structure functions. Different filaments contribute to scaling exponents of different orders. In the model, scaling relations are not produced by singular velocity distribution: there is no singularity at any finite time.

Multifractal formalism is the most adequate approach to describe the observed intermittent behavior of velocity scaling exponents and other quantities of hydrodynamic turbulent flow.
However, the Multifractal approach either implies the existence of singularities or does not specify what structures in a flow, or which solutions of the Navier-Stokes (Euler) equation are responsible for the observed multiscaling properties. In this work we are trying to reveal the mechanism that provides the multifractal scaling.
The subject of the contribution is closely connected with the talk by Zybin, Sirota where it is shown that the requirement of random large-scale component of vorticity in the Navier-Stokes equation results in local exponential stretching of the flow. The regions where initial vorticity is high then become vortex filaments. The flow inside these filaments can be described by one scalar stochastic equation.
Here we consider the simplified linear version of this one-dimensional differential equation:

$$\frac{\partial u(x,t)}{\partial t} + ax\frac{\partial u(x,t)}{\partial x} + bu(x,t) = 0$$

The coefficients $a(t)$, $b(t)$ are random processes corresponding to large-scale oscillations. We demonstrate that the solutions of this stochastic equation provide power law dependence of velocity structure functions and vorticity moments.
The constraints produced by finite viscosity are discussed. It is shown that stationary distribution is possible in some range of scales even if viscosity is taken to be zero. Velocity (and vorticity) field remains smooth at any finite time; the power law is restricted by some finite small scale which decreases itself as a function of time.
We also show that the dispersion of the stochastic large-scale parameters in the equation results in nonlinear dependence of scaling exponents of their orders, i.e., in intermittency. This gives a qualitative explanation of multifractal formalism: different scalings are produced by different vortex filaments. We derive a simple approximation to the dimension function $D(h)$ expressed in terms of the properties of large-scale velocity distribution.