DISPERSION OF PARTICLES FROM LOCALIZED SOURCES IN TURBULENCE

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Abstract We present a detailed investigation of particles relative separation in homogeneous isotropic turbulence. We use data from a $3D$ direct numerical simulations with 1024^3 collocation points and $R_\lambda = 300$ following the evolution of a large number of passive tracers and heavy inertial particles, with Stokes numbers in the range $St \in [0.5, 5]$. Many studies [1, 2, 3] have focused on the subject, including extensions to the case of particles with inertia [4]. In particular, our simulation aims to investigate extreme events characterizing the distribution of relative dispersion in turbulent flows [5, 6]. To do that, we seed the flow with hundred millions of particles emitted from localized sources in time and in space. Thanks to such huge statistics, we are able to assess in a quantitative way deviations from Richardson's prediction for tracers. Furthermore, we present the same kind of measures for heavy particles to understand how the inertia affects the pair separation statistics. Finally, to disentangle the effects of different turbulent scales, we present measurements based on exit time statistics for both tracer and inertial particles.

The relative separation of pairs of fluid particles in turbulent flows was first addressed by Richardson. The main question is simple and fundamental: given a pair of particles released at time t_0 and at a small separation r_0 (smaller of the Kolmogorov dissipative scale η), what is the probability, $P(r,t)$, to find them at a distance r at a later time t?
Richardson proposed to model particle separation in the inertial range $\eta << r << L_0$ as a diffusive process with an effective turbulent diffusivity, estimated empirically to follow a $4/3$ law: $D_{Ric}(r) = k_0 \epsilon^{1/3} r^{4/3}$. Here L_0 is the large scale of the flow, k_0 is a dimensionless constant and ϵ the turbulent kinetic energy dissipation. It is easy to connect Richardson's work with Kolmogorov's theory by means of the dimensional estimate: $D_{Ric}(r) \sim \tau(r)\langle(\delta_r v)^2\rangle$, where $\tau(r)$ is the eddy turnover time at scale r and $\langle(\delta_r v)^2\rangle$ is the second-order Eulerian longitudinal structure function. Richardson's picture captures some important features of turbulent dispersion, e.g. events with a typical separation of the order of the root mean square, $r_{rms}(t) = \langle r(t)^2\rangle^{1/2}$. However, fundamental questions arise on the possibility to correctly predict extremal events, i.e., pairs with separation much larger or smaller than $r_{rms}(t)$.
Richardson's approach can be rephrased as the evolution of tracers in a stochastic Gaussian, homogeneous, incompressible, and isotropic velocity field, δ-correlated in time, with a given two-point longitudinal correlation function $D_{||}(r)$ [7]. Under this assumption, the evolution of $P(r,t)$ is given by:

$$\partial_t P(r,t) = \frac{1}{r^2}\partial_r r^2 D_{\|}(r)\partial_r P(r,t)\,. \tag{1}$$

Whenever the eddy-diffusivity kernel has a power-law behavior, $D_{\|}(r) = D_0 r^{\xi}$ with $0 \le \xi < 2$, the above equation with initial condition $P(r,t_0) \propto \delta(r - r_0)$ can be solved exactly. The Richardson's case is recovered for $\xi = 4/3$.
Tracer behavior in real flows can deviate from Richardson's picture due to several reasons: (i) temporal correlations of the underlying velocity fluid [8, 7, 9], (ii) non-Gaussian velocity fluctuations, (iii) ultraviolet (UV) effects induced by the dissipative subrange, and (iv) infrared (IR) effects induced by a large-scale cutoff. The last two features are connected with finite Reynolds effects [10]. To assess the importance of finite Reynolds number on tracers dispersion in real flows, we have integrated the equation (1) using an effective eddy diffusivity $D_{\|}^{eff}(r)$, that keeps into account both viscous and large-scale behaviors. To do that we have used a fitting formula that reproduces the Eulerian data, and matches the expected UV and IR scaling for both $\tau(r)$ and $\langle(\delta_r v)^2\rangle$ (see [5] for details):

$$\begin{aligned} D_{\|}^{eff}(r) &\sim r^2 & r &\ll \eta \\ D_{\|}^{eff}(r) &\sim r^{4/3} & \eta \ll r &\ll L_0 \\ D_{\|}^{eff}(r) &\sim const. & r &\gg L_0. \end{aligned} \tag{2}$$

The UV and IR cut-offs break the self-similarity of the Richardson assumption and therefore the solution of (1) obtained using $D_{\|}^{eff}(r)$, $P_{eff}(r,t)$, no longer rescales at different times as observed in real turbulent flows (figure (1)). Nevertheless, the far right and left tails still do not match the numerical data, indicating that effects induced by finite-time correlations are probably crucial to predict extreme events in turbulent flows.
In Fig. (2) we contrast the dispersion of two bunches of particles with different Stokes number: effects of inertia are striking. More quantitative results concerning the dependences of the pair-separation on the Stokes number will be presented in the talk.

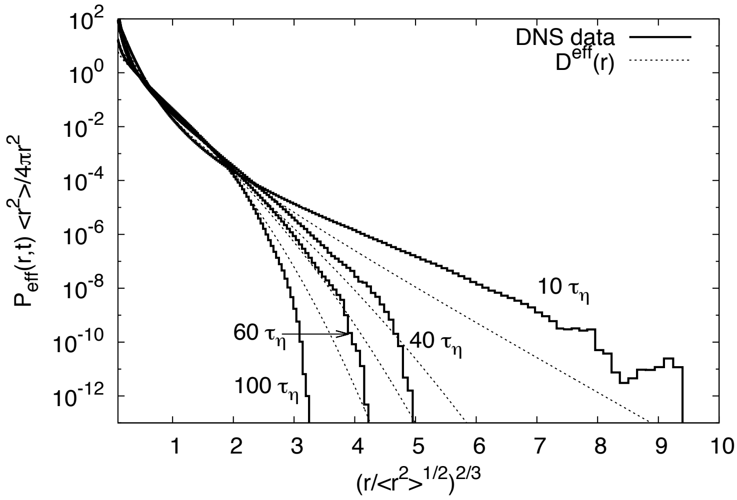

Figure 1. Lin-log plot of probability density function $P_{eff}(r, t)$, obtained from the integration of Richardson's equation with $D_{eff}(r)$ (dashed) and the DNS data (solid line).

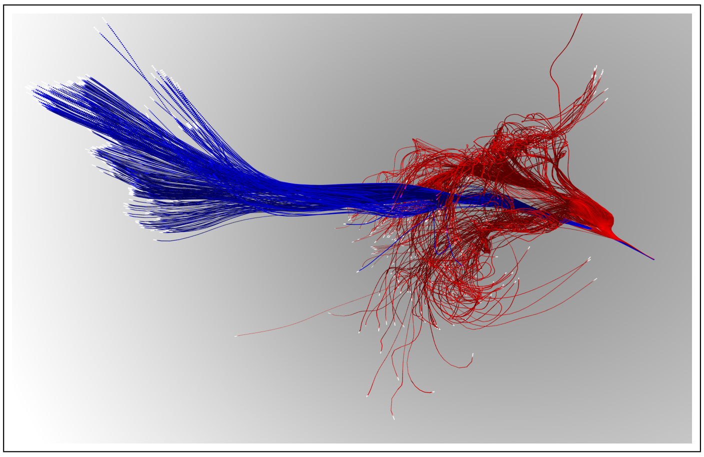

Figure 2. Early time history, up to $t = 75\tau_\eta$, of a bunch of tracers (red) and a bunch of inertial particles with $St = 5$ (blue) emitted from a source of size $\sim \eta$.

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