Experimental observation of a single Lagrangian scale of particle dispersion in developed two-dimensional turbulence

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Abstract This paper reports the Lagrangian statistics in laboratory two-dimensional turbulence in electromagnetically driven turbulence and surface ripple driven turbulence. Our results show that particle dispersion is determined by a single measureable Lagrangian scale related to the forcing scale.

Introduction

The most basic property of Lagrangian trajectories is a single particle dispersion, or mean squared displacement, $\langle \delta r^2 \rangle = \langle |\vec{r}(t) - \vec{r}(0)|^2 \rangle$ of a particle moving along the trajectory $\vec{r}(t)$ from its initial position $\vec{r}(0)$. In turbulence, single particle dispersion is governed by the stochastic equation, and as has been shown by G.I. Taylor[0], it is similar to the Einstein's theory of Brownian motion:

$$\langle \delta r^2 \rangle \approx \langle u^2 \rangle t^2 \,, \qquad \text{at } t << T_L \tag{1}$$

$$\langle \delta r^2 \rangle \approx 2\langle u^2 \rangle T_L t \,, \qquad \text{at } t >> T_L \tag{2}$$

Here u is the particle velocity, $T_L = \int_0^\infty \rho(t)dt$ is the Lagrangian integral time which can be obtained from the Lagrangian velocity autocorrelation function,

$$\rho(t) = \langle u[\vec{r}(t_0 + t)] u[\vec{r}(t_0)] \rangle / \sigma^2 \,, \tag{3}$$

where σ^2 is the velocity variance. To estimate the diffusion coefficient $D = \langle u^2 \rangle T_L$ at large times, one needs to compute, or to measure, Lagrangian velocity correlation function. The problem however is that $\rho(t)$ and T_L cannot be theoretically predicted and their relationship with other flow characteristics is not known.

We report on the measurements of Lagrangian characteristics in two-dimensional (2D) turbulence. Experiments have been conducted in a broad range of the flow kinetic energies, driven at different forcing scales, by using two very different methods of turbulence generation. In the first, turbulence is excited electromagnetically, by the Lorenz force due to the interaction between a square lattice of magnets placed underneath the electrolyte layer and the electric current flowing across the fluid cell[0]. In the second set of experiments, 2D turbulence is driven by Faraday ripples on the surface of a vertically vibrated fluid container [surface ripple driven turbulence(SRT)][3,4].

Experimental Results

The mean squared dispersion of a particle from the initial point on its trajectory is computed by averaging over several thousands of trajectories. The dispersion is shown as a function of time in Fig. 1a for the surface ripple driven turbulence (the forcing scale of $L_f \approx$ 4.4 mm). At shorter times, less than the Lagrangian integral time, $t < T_L$, a clear ballistic regime is observed, $\langle \delta r^2 \rangle \sim t^2$, in agreement with Eq. 1, while at larger times, we find a diffusive regime $\langle \delta r^2 \rangle \approx 2D_{\exp} t$, similar to recent observations[3,5]. The diffusion coefficient $D_{\exp}$, given by half the slope of the dispersion curve, increases when the forcing level is increased (Fig. 2b). This is true in both the surface ripple driven and in electromagnetically driven turbulence. However, when the mean squared dispersions are plotted versus normalized time, t/T_L (Fig. 2c), all dispersion curves of the surface ripple driven turbulence for a given forcing scale collapse onto one. This suggests that the quantity $\tilde{u}^2 T_L^2$ is independent of the turbulence kinetic energy, where $\tilde{u} = \sqrt{\sigma^2}$. It has a dimension of L^2 and this result points to the existence of a universal spatial scale of particle dispersion related to the forcing scale.

To detect this distinct Lagrangian scale we introduce a *spatial* Lagrangian velocity autocorrelation function,

$$\rho(L) = \langle u[\vec{r}(r_0 + L)] u[\vec{r}(r_0)] \rangle / \sigma^2 \,, \tag{4}$$

similar to the temporal velocity autocorrelation function $\rho(t)$ of Eq. 3. This function gives a statistically averaged spatial period between changes in the particle velocity. Moving along a trajectory, we simultaneously record a particle's velocity $u(t)$ and its displacement $\delta r(t)$ from the initial position r_0. The spatial autocorrelation function $\rho(L)$ is computed from measured velocity increments $u(\delta r)$, which are interpolated onto a regular array. The Lagrangian integral scale is determined as $L_L = \int_0^\infty \rho(L)dL$, or, if the autocorrelation function is approximately exponential, $\rho(L) \sim e^{-L/L_L}$, it can be determined from the decay at short distances L.

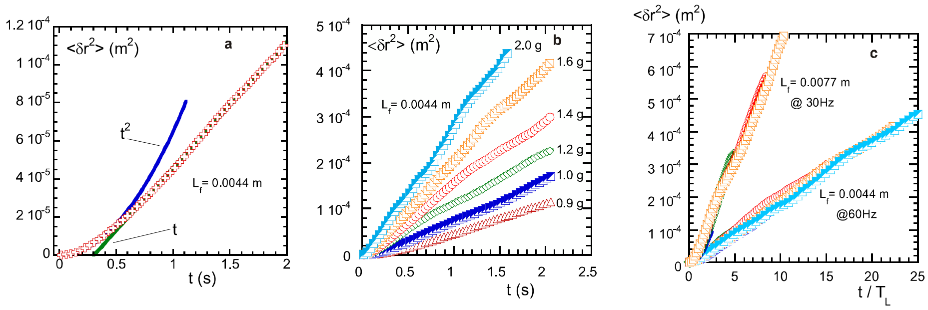

Figure 1. (a) Statistically averaged mean squared displacement of particles away from the initial position. **(b)** Large time mean squared displacements in the surface ripple driven turbulence. **(c)** Mean squared displacement curves at different forcing levels are plotted versus time shift normalized by the Lagrangian integral time.

The Lagrangian velocity autocorrelation time T_L decreases with the increase in energy input (Fig. 2a), however the Lagrangian integral scale L_L is very weakly dependent on energy input. For a given forcing scale in fully developed turbulence spatial Lagrangian velocity autocorrelation functions $\rho(L)$ approximately collapse onto each other, as illustrated in Fig. 2b. In experiments at different forcing scales the Lagrangian integral scale L_L closely follows the scale of forcing. The analysis of all experiments shows that the ratio of the Lagrangian integral scale and the scale of forcing, L_L / L_f, varies in the range from 0.3 to 0.7 in a broad range of conditions. In fully developed turbulence this ratio converges to $L_L / L_f = 0.5 \pm 0.05$. Thus we conclude that particle dispersion in 2D turbulence is determined by the Lagrangian autocorrelation scale and the r.m.s. velocity, $D_{\exp} = \tilde{u} L_L$. Turbulence accumulates energy in the inertial range and determines $\tilde{u}$, while the Lagrangian scale L_L determines the mean free path between the particle's memory loss events. This scale is shorter than any of the energy containing scales in the inertial interval and it is determined by the most persistent scale in the flow, the scale of forcing.

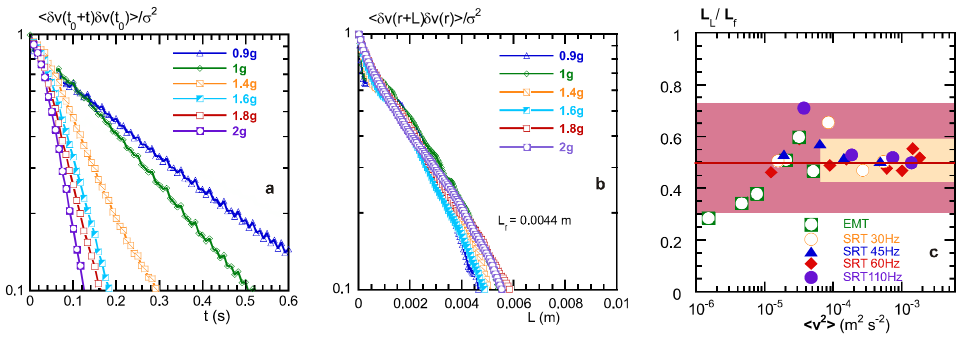

Figure 2. (a) Temporal Lagrangian autocorrelation function, computed for a range of forcing levels in the surface ripple driven turbulence. (b) Spatial Lagrangian autocorrelation function computed for the same conditions. (c) The ratio of Lagrangian integral scale and the corresponding forcing scale L_L / L_f is roughly the same in a very broad range of kinetic energies on the flow and at different forcing scales. In fully developed turbulence this ratio is close to 0.5.

References

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