SPATIOTEMPORAL INVESTIGATION OF CAPILLARY WAVE TURBULENCE: HYPOTHESIS OF WEAK NONLINEARITY UNDER SCRUTINY

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Abstract We report experiments on the full space and time resolved statistics of capillary wave turbulence at the air-water interface. The three-dimensional shape of the free interface is measured as a function of time by using the optical method of Diffusing Light Photography associated with a fast camera. When wave turbulence regime is reached, we observe power-law spectra both in frequency and in wave number, whose exponents are found in agreement with the predictions of capillary wave turbulence theory. However some hypotheses are not really verified as the hypothesis of weak non linearity. We will determine if our observations are compatible with occurrence of 3-wave interactions as expected theoretically. Finally we characterize the spatial and temporal intermittency of the turbulent wave field, and its evolution with wave amplitude.

MOTIVATION

Wave turbulence concerns the study of the statistical properties of a set of numerous waves in nonlinear interaction. For strong enough interactions, a turbulent cascade transfers wave energy from an injection scale towards a dissipation scale, similar to the Richardson-Kolmogorov cascade occurring in hydrodynamics turbulence. Contrary to the classical turbulence, wave turbulence is analytically described in a weakly nonlinear regime by the weak turbulence theory [1]. This phenomenon occurs in various domains of physics: optical waves, surface or internal waves in oceanography, astrophysical plasma waves, Rossby waves ... Several recent studies have tested the relevance of wave turbulence theory in well controlled laboratory experiments for hydrodynamic waves on the surface of a fluid in the gravity and capillary regimes [2]. Most *in situ* or laboratory measurements on wave turbulence involve time signals at a fixed location and show partial agreement with the theory. Spatio-temporal measurements of the turbulent wave amplitude are thus needed to investigate basic mechanisms of wave turbulence. Moreover in oceanography, capillary waves dynamics could control heat and gaseous exchanges between ocean and atmosphere and contribute to the overall dissipation of gravity waves. Accurate characterization of capillary wave turbulence is of primmest interest at fundamental level. According weak turbulence theory, energy transfer through the scales occurs by 3-wave interactions, instead 4-wave interactions for gravity wave turbulence. Therefore, capillary and gravity wave turbulence differ fundamentally. Moreover, the validity domain of wave turbulence theory for capillary waves in experiments remains an open question, due to restrictive hypotheses of the theory (infinite isotropic and homogeneous system, weak non-linearity ...). In particular for scales smaller than or equal to the capillary length, viscous damping of waves is not negligible, whereas theory lies on an Hamiltonian structure of the waves field. Therefore to observe capillary wave turbulence generated by gravity waves, a forcing close to the oceanographic case, a strong wave amplitude is needed, which implies a possibly significant value of the nonlinear parameter inside the capillary cascade. Although weak turbulence theory assumes weakly nonlinear waves so that energy transfer occurs through exact wave resonances, previous temporal measurements [2] and new spatiotemporal ones [3] have shown agreement of power spectra slopes inside capillary range with the theoretical predictions for capillary wave turbulence.

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Capillary wave turbulence is investigated for different forcing amplitudes, in order to reach a strongly nonlinear regime and to determinate its influence on the turbulent cascade. Experiments are performed using a space-time measurement method, the so-called diffusing light photography [4]. This optical method even works for very steep waves contrary to usual optical method based on the local slope measurement. Typically, the surface height is obtained from the transmitted light intensity when a fluid is lighted from below. The fluid is diffusing by adding an amount of micrometer sized spheres (Intralipid 20 %) and scattering of light suppress caustic phenomena. The intensity of the light going through the liquid towards a high definition and high-speed camera is a function of the local height of the fluid. First results have just been obtained showing that this method well works for this problematic with a highly sensitivity [3]. Our experimental device (Fig. 1) consists in a transparent tank ($165 \times 165\,\mathrm{mm}^2$) filled with a diffusing liquid up to a height $h_0 = 30\,\mathrm{mm}$. Surface waves are generated by the horizontal motion of a rectangular paddle driven by an electromagnetic shaker subjected to a random forcing (in phase and amplitude) band-pass filtered in frequency between 4 and $6\,\mathrm{Hz}$. Wave amplitude is spatially characterized by the standard deviation of liquid height σ_h, whereas the degree of nonlinearity is evaluated by using the standard deviation of wave steepness (modulus of local surface gradient) σ_s. Computing the 2D-spatiotemporal power spectrum, we can extract the experimental linear dispersion relation. We have shown that in regimes of wave turbulence, the dispersion relation experiences a nonlinear-shift, possibly related with presence of Stokes waves at large scale (nonlinear gravity waves). However temporal and spatial power spectra (Fig. 2) show a good agreement with theoretical predictions. Moreover by evaluating the potential wave energy, we have shown that the

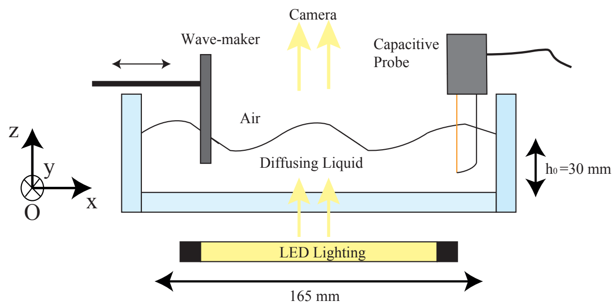

Figure 1. Experimental setup. A LED device ensures a homogeneous lighting below the transparent tank. A fast camera one meter above, is focused on the liquid free-surface. For comparison, the wave amplitude at some location is also recorded by a capacitive wave gauge.

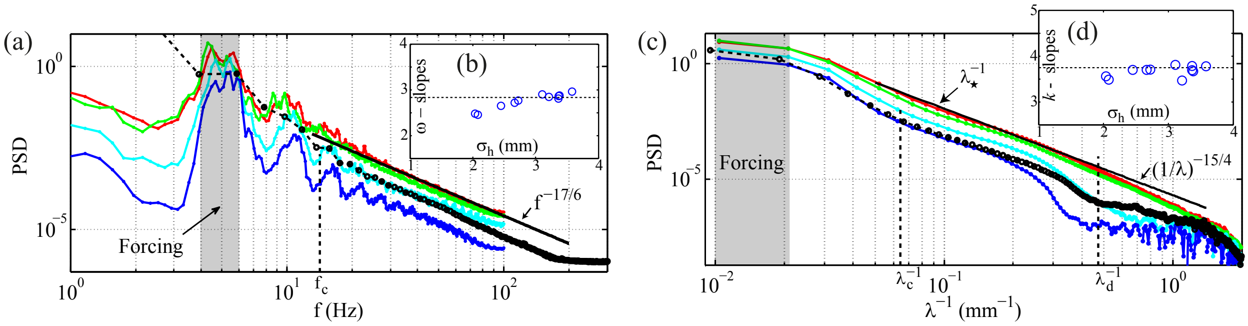

Figure 2. (a) Temporal power spectra $S_h(\omega)$ for different forcing amplitudes. From bottom to top: σ_h =1.3, 2.7, 2.1, 3.4 and 3.6 mm, and $\sigma_s = 0.15$, 0.26, 0.19, 0.3, and 0.34. Solid black line is the capillary prediction $f^{-17/6}$. (b) Inset: $S_h(\omega)$-exponents vs. σ_h (fits from $20 \leq f \leq 100$ Hz). Dashed line shows the theoretical value $-17/6$. (c) Spatial power spectra $S_h(k)$ for the same measurements. Solid black line is the capillary prediction $k^{-15/4}$. (d) Inset: $S_h(k)$-exponents vs. σ_h (fits from $0.094 \leq \lambda^{-1} \leq 0.30$ mm^{-1}). Dashed line shows the theoretical value $-15/4$.

temporal dynamics of the spatial energy spectrum occurs by stochastic bursts transferring wave energy through the spatial scales. The probability distribution function of wave energy evaluated at one wavelength shows when non-linearity is increased a departure from exponential distribution expected for a Gaussian field of random waves. We plan to test the consistency of these observations in wave Turbulence regimes with the mechanisms proposed by the weak turbulence theory. To do this, we will compute high order correlation functions in space and time (bispectrum ...), in order to evaluate occurrence of 3-wave interactions. Then in a strong nonlinear regime, other mechanisms could transfer energy through the scales. In a such regime, we expect to observe microbreakings of waves, capillary bursts and cusps waves (singularities). These events act as coherent structures that break transiently the scale invariance in the turbulent cascade. Presence of these structures and events could be a source of intermittency and could explain observed bursts of wave energy. By means of Wavelets Transform analysis [5], these singular structures could be detected in spatial measurements and their temporal evolution tracked. Finally by computing the statistics of wave height increments, both in time and in 2D space, we will investigate the intermittency of capillary wave turbulence through the scales. Degree of intermittency will be put in relation with presence of coherent structures or occurrence of energy bursts. We expect that the combination of these different analysis tools, will help to elucidate mechanisms at play in capillary wave turbulence and to precise the domain of validity of the theory.

References

[1] V. E. Zakharov, Y.V. Lvov, and G. Falkovich, *Kolmogorov spectra of turbulence* (Springer-Verlag, Berlin, 1992) ; A. Newell and B. Rumpf, Annu. Rev. Fluid Mech. **43**, 59 (2011) ; S. Nazarenko, *Wave Turbulence* (Springer-Verlag, Berlin, 2011).

[2] E. Falcon, Discrete Contin. Dyn. Syst. B **13**, 819 (2010).

[3] M. Berhanu and E. Falcon, submitted to Phys. Rev. E (2012).

[4] W.B. Wright, R. Budakian, and S.J. Putterman, Phys. Rev. Lett. **76**, 4528 (1996).

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