DISPERSION OF A SCALAR PUFF IN TURBULENCE: THEORY AND EXPERIMENT

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Abstract We study the diffusion of a puff of passive scalar in a homogenous and isotropic turbulent flow. We integrate the equation for the spreading of a Gaussian puff using numerical simulations done at Taylor-Reynolds number $\mathrm{Re}_\lambda = 400$. This equation is valid for small puffs, such that the turbulent velocity accross the puff can still be considered linear. The numerical simulations are compared to an experiment in which small Gaussian puffs of NO molecules in a turbulent flow of air are created using molecular tagging. We find striking agreement between simulations and experiment, and discuss the effect of intermittency.

When a Gaussian blob of passive scalar is released in turbulence, it will spread due to the combined action of turbulence and molecular diffusion. When the blob is so small (size $\approx 10\,\eta$) that the local velocity field may be linearized, it is well known that its shape evolves in a Lagrangian frame according to

$$\frac{\mathrm{d}\mathbf{\Gamma}}{\mathrm{d}t} = 4D_m \boldsymbol{I} + \boldsymbol{A}\mathbf{\Gamma} + \mathbf{\Gamma}\boldsymbol{A}^T, \tag{1}$$

where the matrix $\mathbf{\Gamma}$ defines the shape of the Gaussian puff with concentration field $C(\boldsymbol{x}) = \exp(-\boldsymbol{x}^T\mathbf{\Gamma}^{-1}\boldsymbol{x})$, where the elements of the matrix $\boldsymbol{A}$ are the velocity gradients, $A_{i,j} = \partial u_i/\partial x_j$, and where D_m is the molecular diffusion coefficient [1]. We consider the special situation where we release a gaseous puff in turbulent air, so that the kinematic viscosity $\nu \approx D_\mathrm{m}$. A still unanswered question is whether the effect of turbulent dispersion on the spreading of the puff is enhanced or suppressed by the action of molecular diffusion [3].

We study equation 1 with the aid of Lagrangian data coming from highly resolved direct numerical simulations ($Re_\lambda = 400$ with 2048^3 points) [4, 5]. The key results of the simulations can be summarized as follows: (1) the dispersion is dominated by the transverse velocity gradients $A_{i,j} = \partial u_i/\partial x_j, i \neq j$; (2) for short times, the joint action of turbulent dispersion and molecular diffusion can be expressed as an effective diffusion coefficient $\widetilde{D}$, with the largest eigenvalue of $\mathbf{\Gamma}$ evolving as $\Gamma(t) = \Gamma(0) + 4\widetilde{D}t$, with $\widetilde{D} = D_\mathrm{m} + \Gamma(0)(30^{-1/2}\tau_\eta^{-1}$, where the factor $30^{-1/2}$ is related to the dimensional isotropic estimate of the mean transverse velocity gradients.

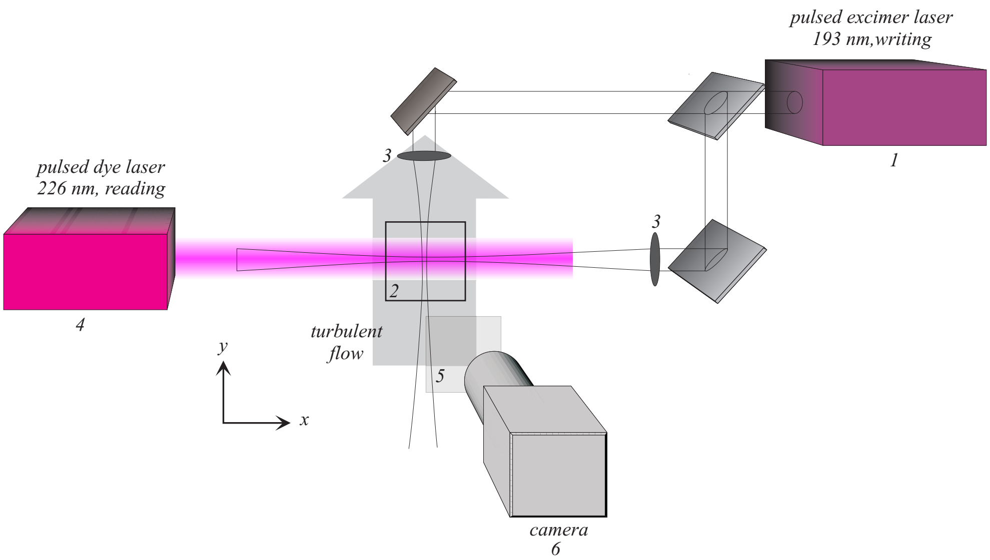

Figure 1. Experimental Setup. The pulsed ArF excimer laser (1) creates a line of NO particles and the flow displaces and wrinkles the lines. The pulsed dye laser (4) visualizes the NO particles a short while later using fluorescence, while the ICCD camera (6) collects fluorescence signals within the readout area (2). Gaussian puffs are realized as lines with Gaussian cross section, or as dots in the intersection point of the two laser beams.

EXPERIMENT

The experimental realization of this problem uses molecular tagging, where small Gaussian pufs were prepared as thin lines (width $\approx 10\ \eta$) using a laser beam with Gaussian cross section, or as small dots in the intersection of two laser beams. These patterns were written in a strongly turbulent jet flow by fusing N_2 and O_2 molecules to NO, which is then used as a tracer. This photosynthesis is done in the focus of strong UV laser beams (ArF excimer Λ Physik, LPX 150). A while (tens of μs) later, the patterns are made visible through inducing fluorescence with a second (UV) laser. The deformed and dispersed pattern of NO molecules is photographed in the UV using a fast intensified camera [2].

In our experiments, the Taylor–based Reynolds number is $\mathrm{Re}_\lambda = 500$, and the Kolmogorov length and time scales are $\eta = 1.4 \times 10^{-5}$ m and $\tau_\eta = 1.4 \times 10^{-5}$ s, respectively. The initial width of the written line is $\sigma_0 = 50\ \mu$m. We trace the backbone $y_0(s)$ of deformed lines in images by fitting Gaussians $I \sim \exp(-(y - y_0(s,t))^2/\sigma^2(s,t))$, to their cross sections and thus find the line width $\sigma(s,t)$, with s the coordinate along the line. From collected statistics on 4×10^3 images at each time delay t we measure how the average line width σ increases with the delay time t between writing and reading.

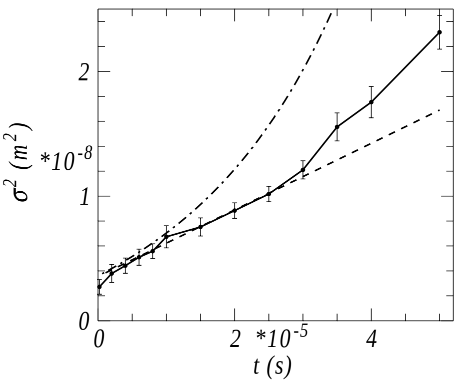

Figure 2. Dots connected by line is the depence of the average width $\langle \sigma^2 \rangle$ on the time delay t between writing and reading lines. The error bars indicate the variation of σ^2 along the written line. Dashed line shows $\langle \sigma^2(t) \rangle = 4\widetilde{D}t$, with $\widetilde{D} = D_\mathrm{m} + \Gamma(0)(30^{-1/2}\tau_\eta^{-1} = 6.8\ \mathrm{m}^2\mathrm{s}^{-1}$. The dash–dotted line indicates $\sigma^2(t) = 4\ D_m\ t + \sigma^2(0)\exp(2\gamma/\tau_\eta)$, with $\gamma = (2/15)^{1/2}$.

The measured Gaussian width σ can be compared to the largest eigenvalue of the matrix $\mathbf{\Gamma}$. The results for delay times t up to $t/\tau_\eta = 3.6$ are summarized in Fig. 2.

CONCLUSION

We find striking agreement between experiment and numerical simulation at short times. These results are for pencil–shaped puffs (1D puffs), but we shall also present results on 3D puffs. A question is whether the strongly anomalous statistics of the gradients $\boldsymbol{A}$ will endow the fluctuations of σ with special properties. A preliminary conclusion is that this is not the case, with the fluctuations of $\sigma/\langle\sigma\rangle$ being close to log–normal.

References

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