

SYMMETRY RELATED SLOW PROCESSES IN PARALLEL SHEAR FLOWS

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Abstract Parallel shear flows, like plane Couette flow or the asymptotic suction boundary layer, come with continuous symmetries of translation in the downstream and spanwise direction. Flow states that differ in their spanwise or downstream location but are otherwise identical are dynamically equivalent. In the case of travelling waves, this trivial degree of freedom can be removed by going to a frame of reference that moves with the state, thereby turning the travelling wave in the laboratory frame to a fixed point in the co-moving frame of reference. Further exploration of the symmetry suggests a general method by which the trivial translational degrees of freedom can be removed also for more complicated and dynamically active states. We will describe the method and explore its implications for the case of the asymptotic suction boundary layer. In the case of the long period oscillatory edge state, the method reveals the slow vortex dynamics underlying the periodic process, and captures the reduction in speed during the bursts. In addition, we find evidence for slow components in the spanwise direction.

SYMMETRIES IN SHEAR FLOWS

Parallel shear flows usually have a continuous translational symmetry in the downstream direction, and often also continuous symmetries in the spanwise direction (if the domains are periodically continued) or in azimuthal direction (in the case of pipe flow). Accordingly, flow states that differ only by a shift in these neutral directions will be dynamically equivalent. This opens up the possibility that processes like the self-regenerating cycle [1] that underlies many dynamical processes in boundary layers may return to a state that is similar to the initial state except for a shift in one of the continuous symmetries. A good example are travelling waves, as found in pipe flow [3, 4]. They are states that are periodic in the laboratory frame, but stationary in a co-moving frame of reference: all that is left of their dynamics is the shift in the downstream direction.

A more complicated situation arises for the case of the asymptotic suction boundary layer, where the edge state intermediate between the laminar and turbulent state shows a periodic process that oscillates between two states that look similar to travelling waves [2]. Now the definition of the speed for the comoving frame of reference is not so obvious, and, as the later calculations show, the speed is also not constant.

SYMMETRY REDUCED EQUATIONS OF MOTION

In order to properly separate the shift in the direction of the continuous symmetry from the time evolution one notes that the shift of a velocity field also introduces a direction of change in the state space of the velocity fields. By projecting the time evolution that follows from the Navier-Stokes equation into components parallel and perpendicular to this symmetry direction one can separate the shift along the symmetry from the proper changes in the velocity field [4]. The method is straightforward to implement in numerical codes.

RESULTS FOR THE ASYMPTOTIC SUCTION BOUNDARY LAYER

When applied to the oscillatory state in the asymptotic suction boundary layer, one notes that speed slows down during the burst. Moreover, the oscillations are connected to the presence of a modulation on top of the downstream vortices and streaks.

In addition to the downstream component one can also study shifts in the spanwise direction. Here one finds fast components, that correspond to high frequency jitter consistent with a homogeneously turbulent state, but also slow components corresponding to drifts in the neutral direction over very long periods (several thousands in natural time units). These could not have been identified without the symmetry decomposition sketched above.

References

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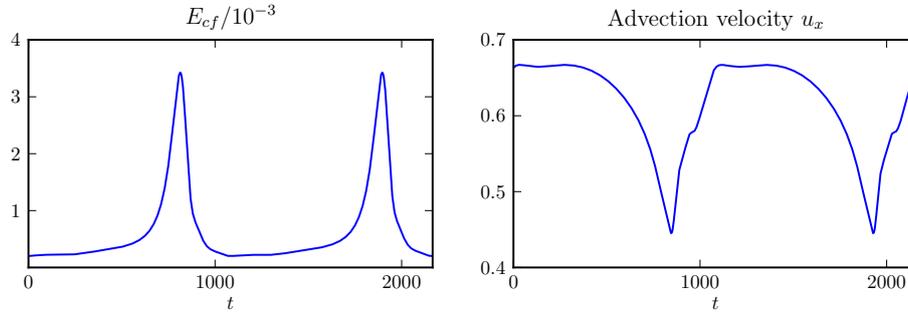


Figure 1. Energy and advection velocity of the edge state in ASBL at $Re = 400$. The left frame shows the clear increase in energy when the edge state goes through the bursting phase. Studies of the flow field then show that afterwards it is displaced by half a box width, so that the full period are two bursting events in energy. The left frame shows the advection speed extracted by symmetry projection. It shows that as the energy goes up the structures slow down noticeably.

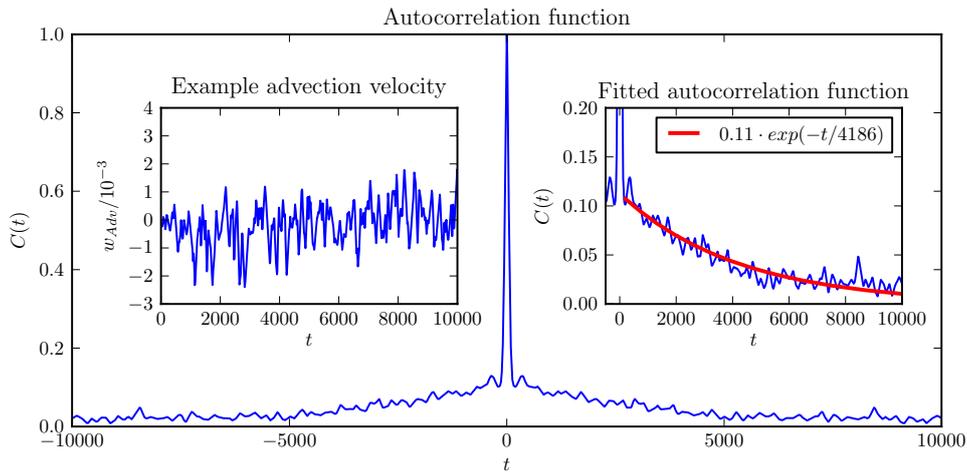


Figure 2. Statistics of transverse shift velocities determined by symmetry projection onto the span wise degrees of freedom. The inset shows that the velocities are small and fluctuate randomly. The correlation function (main frame) shows a strong central peak, indicative of weak correlations, and a broad tail reaching out to very long times. The right inset shows that this tail is well captured by an exponential distribution with a characteristic time of more than 4000 natural units.