BLENDED SCALE-SEPARATION MODELS FOR LARGE EDDY SIMULATIONS

Roel Verstappen
Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, The Netherlands

Abstract At the crossroads of theory and simulation new dissipation and regularization models for large eddy simulation (LES) start to develop. The basic idea is that the LES-solution should contain only scales of size $\geq \Delta$, where Δ is a user-chosen length scale. In case the convective nonlinearity produces scales $< \Delta$ (forward scatter) we compute an eddy viscosity such that the resulting eddy diffusion (counter)balance the production term approximately. In case of backscatter, the scale seperation condition is satisfied by regularizing the nonlinear production. The resulting blend of dissipation and regularization is successfully tested for turbulent channel flow (Re_τ=590).

Large-eddy simulation (LES) seeks to predict the dynamics of spatially filtered turbulent flows. Therefore a spatial filter $u \mapsto \overline{u}$ is applied to the (incompressible) Navier-Stokes (NS) equations. Additionally we regularize the convective nonlinearity with the help of second filter $u \mapsto \tilde{u}$:

$$\partial_t \overline{u} + \widetilde{(\tilde{\overline{u}} \cdot \nabla)}\tilde{\overline{u}} + \nabla \overline{p} - \nu \nabla^2 \overline{u} = \nabla \cdot (\widetilde{\tilde{\overline{u}} \otimes \tilde{\overline{u}}} - \overline{u \otimes u}), \qquad (1)$$

The right-hand depends on both u and $\tilde{\overline{u}}$, due to the nonlinearity. The dependence on u is removed by introducing a closure model. Here we take an eddy viscosity model. The motion of the larger eddies is then governed by

$$\partial_t v + \widetilde{(\tilde{v} \cdot \nabla)}\tilde{v} + \nabla q = \nabla \cdot (\nu + \nu_t) S(v) \qquad (2)$$

where $S(v)$ denotes the symmetric part of the velocity gradient. The variable name is changed from $\overline{u}$ to v to stress that the solution of Eq. (2) differs from that of Eq. (1), because the closure is not exact. A lower bound for the eddy viscosity ν_t is determined from the requirement that the production of any eddies of size smaller than Δ by the nonlinear mechanism in the left-hand side of Eq. (2) is counteracted by the eddy dissipation in the right-hand side of Eq. (2). To that end, we consider an arbitrary part Ω_Δ with diameter Δ of the flow domain and take the filtered velocity $\overline{u}$ equal to the average of u over Ω_Δ (box filter). The Poincaré inequality states that

$$\int_{\Omega_\Delta} ||v - \overline{v}||^2 \, dx \;\leq\; C_\Delta \int_{\Omega_\Delta} ||\nabla v||^2 \, dx,$$

for every v, where the constant is given by $C_\Delta = (\Delta/\pi)^2$. The residual field $v' = v - \overline{v}$ contains eddies of size smaller than Δ. The eddy viscosity must keep them from becoming dynamically significant. Poincaré's inequality shows that this can be achieved by damping the velocity gradient. According to Eq.(2) the $L^2(\Omega_\Delta)$ norm of ∇v dissipates (at least) at its natural rate, that is

$$\frac{\mathrm{d}}{\mathrm{dt}} \int_{\Omega_\Delta} \tfrac{1}{2} ||\nabla v||^2 \, dx \;\leq\; -\nu \int_{\Omega_\Delta} ||\nabla^2 v||^2 \, dx,$$

if the eddy viscosity ν_t is taken such that

$$\nu_t \int_{\Omega_\Delta} q(v) \, dx \;\geq\; C_\Delta \int_{\Omega_\Delta} r(\tilde{v}) \, dx, \qquad (3)$$

where q and r are the invariants of the rate-of-strain tensor S: $q(v) = \frac{1}{2}\mathrm{tr}(S(v)^2)$ and $r(\tilde{v}) = -\det(S(\tilde{v}))$. Without the regularization the right-hand side depends on $r(v)$; see [1]-[2] for a mathematical derivation. The regularization in (1)-(2) is taken in such in way that we get $r(\tilde{v})$ instead of $r(v)$. Condition (3) ensures that the transfer of energy *from* the large eddies to the subfilter scales is balanced properly by the eddy dissipation. To limit the dynamics governed by Eq. (2) to scales of size $\geq \Delta$, the energy that is transferred from the subfilter scales *to* the large eddies, should be dynamically insignificant too. This second scale separation condition is imposed by setting $r(\tilde{v}) = 0$ if $r(v) < 0$. Notice $r(v) < 0$ corresponds to backward transfer of energy to the larger eddies. To that end, we consider the following, generic filter,

$$\tilde{v} = v - \tfrac{1}{24}\epsilon^2 \nabla^2 v$$

Here the filter length ϵ is taken such that $r(\tilde{v}) = 0$, that is ϵ is to be solved from the generalized eigenvalue problem

$$\det\left(S(v) - \tfrac{1}{24}\epsilon^2 S(\nabla^2 v)\right) = 0$$

Obviously, we take the smallest non-negative eigenvalue (existence can be proven). Note if $r(v) > 0$ then $\epsilon = 0$.

The minimal eddy viscosity satisfying the scale separation condition (3) becomes

$$\nu_e(v) \;=\; C_\Delta \, \frac{\overline{r(\tilde{v})}}{\overline{q(v)}} \tag{4}$$

Now the problem is that we need know how r and q vary within Ω_Δ to compute $\overline{r(\tilde{v})}$ and $\overline{q(v)}$. Here, we cannot simply take $\overline{q(v)} = q(\overline{v})$, because the relation between q and v is nonlinear (similarly for r). This problem has a likeness with the closure problem in LES, except that the original closure problem concerns the residual of the Navier-Stokes solution u, whereas here it is about the residual of the large-eddy solution v. We apply an approximate deconvolution method that recovers some of the information lost in the filtering process, see [3], e.g. To recover an approximation for v' we consider the series expansion of v around $\overline{v}$. Ignoring terms that are of the order Δ^4, we get the approximation $v' \approx -\frac{1}{24}\,\Delta^2\;\nabla^2\overline{v}$. Thus, we arrive at the (second-order) approximation

$$\nu_t(v) \;=\; \tfrac{3}{2}\; C_\Delta \; \frac{r(\tilde{v})}{q(v)} \tag{5}$$

This eddy viscosity model has the following properties: (a) $\nu_t = 0$ in any (part of the) flow where $r \leq 0$, i.e., the eddy viscosity vanishes if the nonlinear transport to scales $< \Delta$ is absent; (b) $\nu_t = 0$ in all flows in which it should vanish according to Vreman [4]; (c) $\nu_t = 0$ at a wall; (d) $\nu_t \to 0$ if $\Delta \propto Re^{-3/4}$; and (e) the corresponding Smagorinsky coefficient is bounded by Lilly's value: $C_S \leq 0.17$. It goes without saying that the performance of the eddy viscosity model (5) has to be investigated for many cases. As a first step it was tested for turbulent channel flow by means of a comparison with direct numerical simulations of Moser *et al.* [5] at Re_τ = 590. The computational grid used for the large-eddy simulation consists of 64^3 points. Unlike the standard Smagorinsky model (even with the relatively low value $C_S = 0.1$), the present model showed an appropriate behavior. As can be seen in Fig. 1 both the mean velocity and the root-mean-square of the fluctuating velocity are in good agreement with the DNS. To illustrate how much the eddy viscosity model contributes to the quality of the solution, the mean velocity profile obtained on the 64^3 LES-grid without closure model (i.e., $\nu_e = 0$) is also shown in Fig. 1.

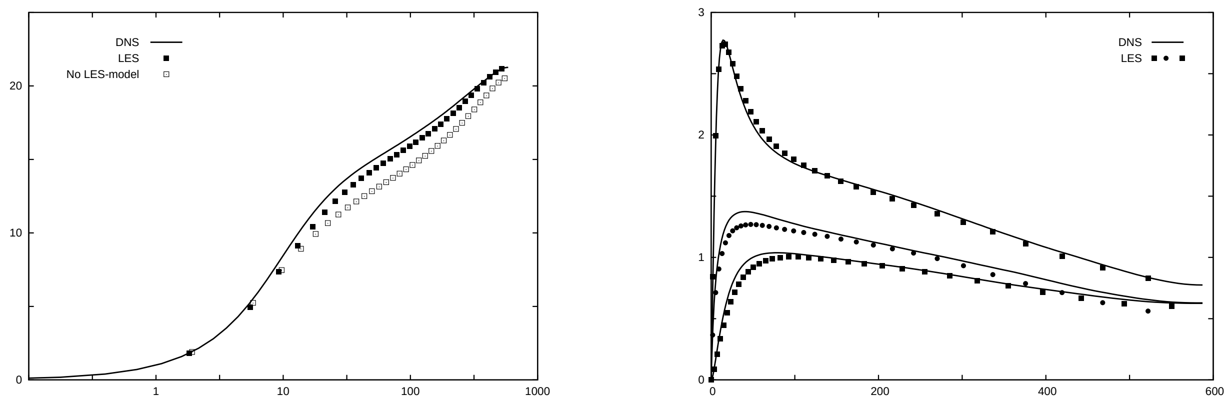

Figure 1. The left-hand figure shows the mean velocity (in wall coordinates) obtained with the help of the 64^3 LES and the DNS by Moser et al. [5]. Results obtained on the 64^3 LES-grid without closure model (i.e., $\nu_e = 0$) are also shown for reference (open symbols). The right-hand figure displays the root-mean-square of the fluctuating velocities. The boxes and circles represent LES data; every symbol corresponds to data in a grid point.

References

[1] Verstappen, R.W.C.P., Bose, S.T., Lee, J., Choi, H. and Moin, P. (2011). A dynamic eddy-viscosity model based on the invariants of the rate-of-strain. Proceedings of the Summer Program 2010, Center for Turbulence Research, Stanford University, 183–192.

[2] Verstappen, R. (2011). When does eddy viscosity damp subfilter scales sufficiently?, J. Sci. Comput. 49: 94–110.

[3] Berselli, L.C., Illiescu, T. and Layton, W.J. (2006). Mathematics of large eddy simulation of turbulent flows, Springer, Berlin Heidelberg New York.

[4] Vreman, B. (2004). An eddy viscosity subgrid-scale model for turbulent shear flow: Algebraic theory and applications Phys. Fluids 16, 3670.

[5] Moser, R.D., Kim, J. and Mansour, N.N. (1999). Direct numerical simulation of turbulent channel flow up to Re_τ=590. Phys. Fluids 11:943–945.