PARTICLE CLUSTERING IN RADIATION-INDUCED TURBULENCE

Rémi Zamansky[1], Filippo Coletti[2] Marc Massot[1,3] & Ali Mani [1,2]
[1]*Stanford University, CTR, USA*
[2]*Stanford University, Mechanical Engineering, USA*
[3]*Ecole Centrale Paris, EM2C - UPR CNRS 288, France*

Abstract In the context of flows laden with inertial particles, we explore a novel regime, in which turbulence is sustained solely by the thermal radiation absorbed by the dispersed phase. When a fluid laden with particles is subject to radiative flux, non-uniformities in particle distribution result in local temperature fluctuations. Under the influence of gravity, buoyancy induces the vortical fluid motion leading to higher non-uniformities of inertial particles. From numerical simulations it is shown that the feedback loop between the local particle concentration, the temperature fluctuations and the convective motion can create and sustain turbulence. When the particle response time is comparable to the temporal scales of the flow, the system exhibits intense fluctuations of turbulent kinetic energy associated to the high intermittency of the particle concentration.

The proposed study is aimed to understand the interplay of hydrodynamic turbulence, radiative heating and particle transport. These three fundamental phenomena are encountered simultaneously in many branches of physical science, from meteorology to engineering, oceanography and astrophysics. In fluid flows laden by particles or droplets, the velocity lagging of the dispersed phase with respect to the carrier fluid can lead to significant local concentration in zones of shear and away from vorticity cores [2].

In this study, we consider a homogeneous suspension of particles subject to radiative heat flux in presence of a gravity field. Non-uniformities in particle concentration result in temperature variations, due to the different absorptivity of the dispersed and carrier phases. Fluid motion is induced by buoyancy forcing. The resulting baroclinic vorticity production induces higher non-uniformities in the particle distribution. The coupling between local particle concentration, temperature fluctuations and hydrodynamic forcing results in a spontaneous and self-sustained feedback loop that can trigger and sustain turbulence in a large fraction of the parameter space.

We focus here on the weak radiative flux regime. In this limit, we can assume that the temperature of the system is quasi-stationary, and that the density variation is small enough to be retained only in the buoyancy forcing term. In line with these assumptions, the governing equation of the carrier phase are obtained in the framework of the Oberbeck-Boussinesq approximation [1]. They read:

$$\nabla.\mathbf{u} = 0 \ , \qquad D_t\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u}^+ + g\alpha\theta\mathbf{e_z} \ , \qquad D_t\theta = \kappa\nabla^2\theta + \frac{q'}{\rho c_f} \tag{1}$$

where $D_t = \partial_t + \mathbf{u}.\nabla$, ν is the kinematic viscosity, κ is the thermal diffusivity, $\theta = T - T_0$ is the temperature fluctuation around the reference temperature, α is the isobaric thermal expansion coefficient, c_f is the fluid heat capacity, g is the gravity, and $q' = q - \overline{q}$ represents the spatial fluctuations of the thermal source term, with $\overline{q}$ the total heat flux absorbed by the system per unit volume. These equations are solved using a pseudo-spectral method in a periodic cubic domain. For the particle phase, we use the Lagrangian approach to obtain the evolution of the particle velocity and position. It is assumed that the particles present a much higher density than the fluid and are much smaller than the computational mesh. It is then legitimate to consider the particles as material points. The evolution equations for a particle are then given by:

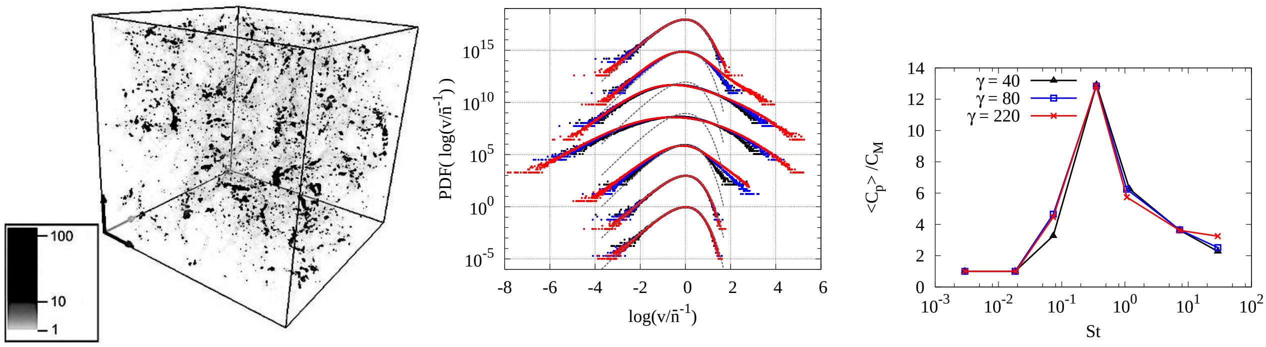
$$d_t\mathbf{x}_p = \mathbf{u}_p \ , \qquad d_t\mathbf{u}_p = \frac{\mathbf{u} - \mathbf{u}_p}{\tau_p} + \frac{\rho_p - \rho}{\rho_p} g\mathbf{e_z} \tag{2}$$

$\mathbf{x}_p$ is the particle position coordinate, $\mathbf{u}_p$ is the particle velocity, ρ_p is the particle density, m_p is the mass of a particle, and τ_p is the particle relaxation time. Φ_p represent the radiative heat flux absbored by one particle, and it can be related to $\overline{q}$: $\Phi_p = \overline{q}/\overline{n}$, with $\overline{n} = N_p/H^3$ the mean particle number density, H the size of the computational domain and N_p the number of particle in this box. The gas velocity at the particle position is estimated from cubic spline interpolation [3]. The physics is further simplified by considering the particles in thermal equilibrium with the surrounding fluid. Then the inter-phase heat exchanges are obtained by projection of the Lagrangian quantities onto the Eulerian mesh: $q(\mathbf{x}) = \sum_p^{N_p} \Phi_p \delta(\mathbf{x} - \mathbf{x}_p)$ where, similarly to [4], we applied a Gaussian regularization to approximate the Dirac distributions of the source term.

Since the forcing of the flow is related to the particle segregation, we would expect the key parameter to be the particle Stokes number St, *i.e.* the ratio of the particle response time to the time scale of the flow structures [2]. However, unlike in "one-way-coupled" particle-laden flows (where the fluid flow is not influenced by the dispersed phase), here the flow time scale is not known a priori. Nevertheless, an expression for the flow time scale, t_*, is derived by dimensional analysis. The size of the box, H, is assumed to be not directly relevant for the particle segregation, and τ_p is not retained in order to build a non-dimensional parameter corresponding to the Stokes number. Dimensional analysis yields $t_* = (\alpha g \beta)^{-2/5} \nu^{1/5}$ where $\beta = \dfrac{d\langle T \rangle_s}{dt} = \dfrac{\overline{q}}{\rho c_f}$ is the mean rate of fluid temperature increase. The temperature scale is set by imposing $\theta_* = \beta t_*$, and the length scale, obtained from the Brunt-Väisälä frequency $t_*^{-1} = (\alpha g \theta_* / \ell_*)^{1/2}$, is $\ell_* = (\alpha g \beta)^{-1/5} \nu^{3/5}$. In connection with equations (1) and (2), the non-dimensional form of the set of parameters can be expressed as: the Stokes number $St = \tau_p / t_*$, the Reynolds number (or a confinement parameters) $\gamma = H/\ell_*$, the density and heat capacity ratios ρ_p/ρ_f the Prandtl number $Pr = \nu/\kappa$, the Froude number $Fr = \left(g t_*^2 / \ell_*\right)^{-1/2}$ and the non-dimensional particle number density $C = \overline{n}\ell_*^3$.

We have run a set of simulations for 7 Stokes numbers (ranging from 3×10^{-3} to 30) and 3 Reynolds numbers ($\gamma = 40$, 80, 220), keeping all other parameters constant. ($Pr = 1$, $\rho_p/\rho = 909$, $1/Fr = 0$ and $C = 0.35$) These simulations correspond to $N_p = 2.31 \cdot 10^4$, $1.10 \cdot 10^5$ and $2.00 \cdot 10^6$ particles, respectively, in a domain of size $(2\pi)^3$ with computational mesh of 65^3, 128^3 and 256^3 elements, respectively. This ensure that all the physical scale of the flow are properly resolved and the particle diameter much smaller than the grid size. All simulations are initiated with quiescent flow conditions (zero velocity and temperature fluctuations) and a random distribution of particle.

After an initial spin up, the system reach a statistical steady-state. The influence of the particle response time on the dynamics is demonstrated in [6]. In particular the variance of the fluid temperature as well as the turbulent kinetic energy of the system present a sharp pic around $St \approx 1$. As seen in the flow visualization snapshots, this peak is connected to a very high particle segregation. We base our analysis of the particle clustering on Voronoi tessellation of the particle positions [5]. The PDF of the volume of the Voronoï cells (see the figure of the middle) present very stretched tails for intermediate St which is a manifestation of the high intermittency of the particle distribution. The sets of particles that define the clusters are determined using the connectivity of the Voronoï cells. Based on this cluster definition it becomes possible to compute statistics characterizing the clusters. For example, the figure on the right hand side presents the evolution with the Stokes number of the particle concentration in cluster average over all the clusters. Furthermore by introducing a cluster-cluster correlation we can track the temporal evolution of clusters and detect the clusters merging and splitting.



(left): Snapshot for $\gamma = 80$ and $St = 0.07$ of the particle concentration (compared to the mean concentration). (middle): PDF of the volume of the Voronoï cell, for $St = 0.003$, 0.019, 0.074, 0.352, 1.064, 7.343 and 29.36 (respectively shifted upward), for $\gamma = 40$ (black), 80 (blue) and 220 (red), comparison with the PDF (in gray) for the Poisson distribution. (right): Mean particle concentration in cluster versus the Stokes number for $\gamma = 40$ (black), 80 (blue) and 220 (red).

References

[1] Vadim Borue and Steven A. Orszag. Turbulent convection driven by a constant temperature gradient. *Journal of Scientific computing*, **12**(3), 1997.

[2] J. K. Eaton and J. R. Fessler. Preferential concentration of particles by turbulence. *International Journal of Multiphase Flow*, **20**:169–209, 1994.

[3] R. Garg, D. Narayanan, C. Lakehal, and S. Subramaniam. Accurate numerical estimation of interphase momentum transfer in lagrangian–eulerian simulations of dispersed two-phase flows. *Int. J. of Multiphase Flow*, **33**:1337–1364, 2007.

[4] M R Maxey, B K Pate, E J Chang, and L-P Wang. Simulations of dispersed turbulent multiphase flow. *Fluid Dynamics Research*, **20**:1–6, 1997.

[5] R. Monchaux, M. Bourgoin, and A. Cartellier. Preferential concentration of heavy particles: a Voronoi analysis. *Physics of Fluids*, **22**:103304, 2010.

[6] Rémi Zamansky, Filippo Coletti, Marc Massot, and Ali Mani. Buoyancy-driven turbulent flow in particle-laden fluid subject to radiation. In *Center fot Turbulence Research Annual Research Briefs 2012*, Stanford University, California, 2013. Center for Turbulence Research.