TURBULENCE INDUCED COARSENING ARREST IN SPINODAL DECOMPOSITION

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Abstract We study the spinodal decomposition of a binary mixture in presence of fluid turbulence. Our study shows that the coarsening is arrested in presence of turbulence. The scale at which the coarsening is arrested can be estimated by means of the phenomenological argument proposed by Hinze [4] to estimate the maximum stable droplet diameter in turbulent emulsions.

INTRODUCTION

Turbulence is ubiquitous and is known to strongly enhance mixing and transport in fluids. A scalar dispersed in a turbulent flow continuosly undergoes stretching and folding and mixes on time and spatial scales much shorter than what a purely diffusive mixing would predict. The enhanced mixing of turbulence can be associated to the fact that it excites fluctuations at all the length and time scales and is understood, by means of simple dimensional arguments, in terms of an scale-dependent eddy-viscosity $\nu_t(\ell) \sim \nu(\ell/\eta)^{4/3}$ [12] (for inertial scales $\ell > \eta$, and hence $\nu_t > \nu$).
When a binary mixture is cooled below its critical temperature it undergoes a phase transition and the mixture separates into its individual components. This phenomenon is widely known as spinodal decomposition. Theoretically, the temperature below which the system undergoes the phase transition is determined by finding out the point where the free-energy minimum becomes degenerate. The dynamics of the phase separation can be modeled by means of the Navier-Stokes equations coupled to a Cahn-Hilliard or model-B equations [1, 6]. Using dimensional estimates, the evolution of the phase separation can be divided into three regimes. a) The initial state: the coarsening length scale grows as $t^{1/3}$ (referred as *Lifshitz-Slyozov scaling*, [8]). This corresponds to a growth dominated by the binary mixture diffusivity and is associated with evaporation-condensation mechanism. b) At intermediate times: viscous dissipation of the fluid balances the pressure ($\nu\nabla^2\boldsymbol{u} \sim \nabla p$) and leads to linear with t increase in the coarsening length (referred as *Viscous scaling*, [14]). c) At final stages: the coarsening length scale grows as $t^{2/3}$ and is governed by the balance of fluid advection with the

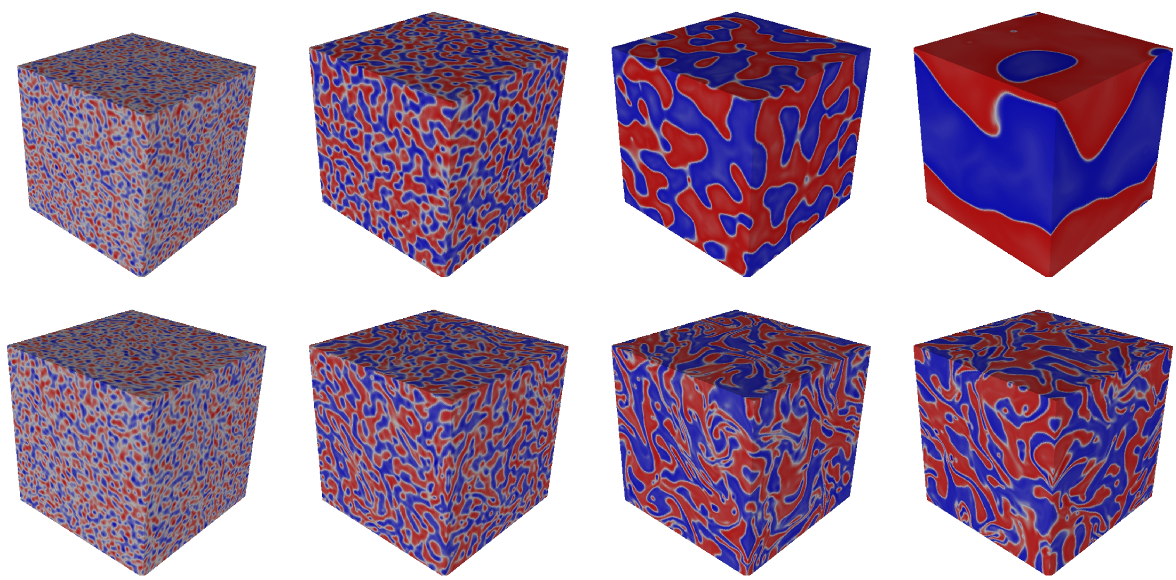

Figure 1. Pseudo-color plots of the concentration fields for coarsening with and without turbulence. (Top panel, left to right) Time evolution, from left to right, of the concentration field undergoing the coarsening process. The formation of larger domain sizes is clearly visible as time progresses. (Bottom panel, left to right) Time evolution, from left to right, of the concentration field undergoing a coarsening process in presence of turbulence generated by an external stirring mechanism. The coarsening process proceeds similarly in the early phases until the turbulence level are strong enough to stop the growth of the concentration domains. The snapshots are taken at times $t = 5.0 \cdot 10^3, 1.0 \cdot 10^4, 2.5 \cdot 10^4$, and $1.0 \cdot 10^5$. In presence of turbulence the coarsening of the concentration gets arrested while in absence of turbulence the growth continues until it reaches the domain size.

variations in chemical potential ($\rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \nabla \mu$) (referred as *Inertial scaling*, [2]). This evolution of the coarsening process has been verified in earlier numerical [9, 7, 10, 3] and experimental studies [5]. Here, we study the interplay between turbulence and the coarsening dynamics coarsening. Turbulence twists, folds, and breaks interfaces into smaller domains whereas coarsening leads to merging of domains.

METHODS AND RESULTS

Our numerical investigations employ a Lattice Boltzmann method based on the standard Shan-Chen algorithm for multi-component fluids [13]. The initial configurations were generated with random, white-noise in space density fields for the two components. The large-scale turbulent stirring was implemented in real space as a sum of sine-waves with phases evolved according to Ornstein-Uhlenbeck processes with relaxation times of the order of the turbulent integral time scale. The dynamic evolution of the coarsening process can be observed in Figure 1 (top row: without turbulence; bottom row: with turbulent stirring).

In Figure 2 we show that as a result of the competing mechanisms of turbulence and spinodal decomposition the coarsening process gets arrested. In the inset of Figure 2 we show that the arrest length-scale can be estimated be the classical phenomenological argument of Hinze for droplet stability in turbulent emulsions [4, 11]. Remarkably the phenomenological argument for single droplet stability in diluted turbulent emulsion can be employed in the dense regime of a 50%-50% multicomponent mixture.

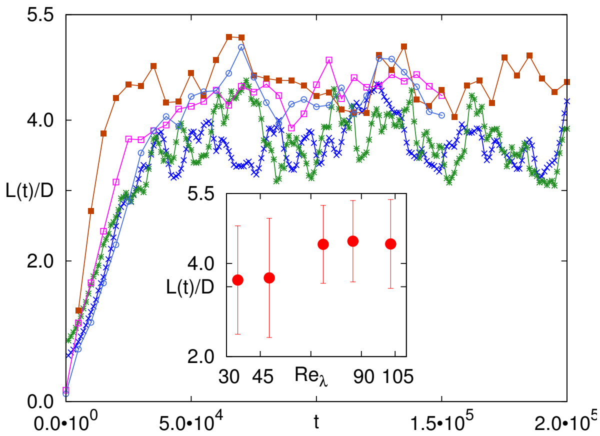

Figure 2. Growth of the coarsening length scale, $L(t)$, as a function of time and normalized by the Hinze estimate, D, for $Re_\lambda \sim 35$ and 49 (blue cross and green star), $Re_\lambda \sim 72$ and 103 (purple square and brown filled square), and $Re_\lambda \sim 86$ (blue circle). Inset: average value of $L(t)/D$ for different Re_λ.

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