RESTORING ISOTROPIC UNIVERSALITY IN FREELY DECAYING ROTATING TURBULENCE

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Abstract We analyse the anisotropy of homogeneous turbulence in a rotating frame. The Zeman scale k_Ω (Zeman, 1994) was introduced to quantify the effect of non-linearity compared to the Coriolis effect, and quantify anisotropy at different scales: at large scales $k \ll k_\Omega$ the anisotropy created by rotation is dominant, whereas at small scales $k_\Omega \ll k$, universal 3D isotropic characteristics of turbulence appear to be restored. We investigate the corresponding phenomenon using Direct Numerical Simulations (DNS) in freely decaying turbulence, varying the rotation rate. We confirm the return to isotropy of the small scales by analyzing the angular power spectrum, the energy transfer and poloidal/toroidal energy modes. The universality is restored at small scales, but for the larger rotation rate case.

CONTEXT

In astrophysical, geophysical and engineering flows, turbulence can be strongly affected by solid-body rotation. In contrast with isotropic turbulence, some anisotropic structuration emerges in rotating turbulence. These structures are elongated along the rotation axis, and consist of both vortices and jets, characterized as “2D-3C" — two-dimensional, three-component —. Two-dimensional trend means a reduction of axial variability, but generally not reduced to pure Taylor columns. This complex anisotropy requires a refined statistical description, with two-component spectra (see results here) or two-component structure functions [3]. The question is whether this anisotropy, characteristic of rotating turbulence, is present at all scales? To answer this question and provide a measure of the anisotropisation phenomenon, Zeman [1] introduced a wavenumber cut-off scale k_Ω defined as

$$k_\Omega = \left(\Omega^3/\epsilon\right)^{1/2} \tag{1}$$

with Ω the rotation rate and ϵ the energy dissipation rate. This number k_Ω, analogous to the Ozmidov scale in stratified flows, is called the Zeman scale; it quantifies the effect of non-linearity compared to the anisotropic effect of rotation relative to different scales: at large scales $k \ll k_\Omega$ the anisotropy is dominant whereas at small scales $k_\Omega \ll k$ the universal 3D isotropic characteristic is restored.

One recent DNS [2] seems to confirm this description. However, in these simulations, only a short Coriolis subrange appears and only one value is given to the rotation rate. Moreover, these are forced simulations, with a forcing term which is anisotropic at large scales so it is in competition with the natural rotation anisotropy of large scales. From data obtained in a very nice recent experiment by [3], in which a measure of the complete anisotropic energy transfers is done, the estimation of k_Ω also permits to say that, to this day, experimental PIV techniques cannot reach a high enough resolution to capture the Zeman scale.

Therefore, in order to clarify the small-scale isotropisation phenomenon, we present some results of DNS with six different rotation rates, considering the case of freely decaying rotating turbulence.

RESULTS

Classically, to measure the energy by scale — or for each wave number in Fourier space —, one uses averages of energy over spheres of radius k, and thus averages out the anisotropic contents of the energy distribution. In the case of rotating turbulence with axisymetric statistics about the rotating vertical axis, the distribution of energy is not equi-distributed over the spherical shell of radius k by contrast to isotropic turbulence. We characterise this non equi-distribution of kinetic energy by introducing the angular dependence of the power spectrum ([4] and references therein). In the case of discrete analysis in DNS, we decompose the sphere into several rings O_i (six rings in our simulation as shown on the sketch of figure 1):

$$E(k, O_i) = \frac{1}{m_k^i} \sum_{\mathbf{k}\in O_i} |\hat{\mathbf{u}}_\mathbf{k}(t)|^2$$

where $\hat{\mathbf{u}}_\mathbf{k}$ is the Fourier velocity vector, and $m_k^i = \frac{\pi}{2}\frac{1}{2(\theta_i-\theta_{i+1})}\frac{1}{(sin(\theta_i)-sin(\theta_{i+1}))}$ is a normalization term.

We have performed simulations at six rotation rates, thus at different Rossby numbers ranging from about 0.03 to 0.3, at two differents resolutions: 1024^3 and 2048^3 points. For example, on figure 1 we plot the power spectrum for four rotation rates and the Zeman scale k_Ω in DNS with 1024 points.

We show that at high Rossby number (low rotation rate), on figure 1(a), all scales have a universal 3D isotropic characteristic (every angular spectrum power $E(k, O_i)$ collapse) and, since the Zeman scale k_Ω is larger than all the turbulent vortices in the flow, rotation has no effect. Nevertheless, at lower Rosby number (higher rotation rate), on figures 1(b) and (c), the large scales exhibit an anisotropy in which energy concentrates on the equatorial ring O_8 down to the Zeman scale $k \leq k_\Omega$. Below this Zeman scale, *ie.* $k \geq k_\Omega$, the scales characteristics recover a universal isotropic distribution (every angular spectrum power collapse again). Finally, at very high rotation rate, on figure 1(d) (very low Rosby number), every scale presents anistropic characteristics.

In our presentation, we will discuss more precisely this phenomen, in particular with DNS results at high resolution 2048^3. We will show that the non linear energy transfer is subjected to the same phenomenon. Moreover, by decomposing the Fourier velocity vector in a poloidal part (aligned to the rotation axis) and a toroidal part (perpendicular to the rotation axis) [4], the componements of velocity in the inertial range follow the scenario of "2D-3C" — two dominant directions of variability but still retaining three-components velocity — at low Rosby number at small scales and universal 3D isotropic characteristic at high Rosby number for every scale.

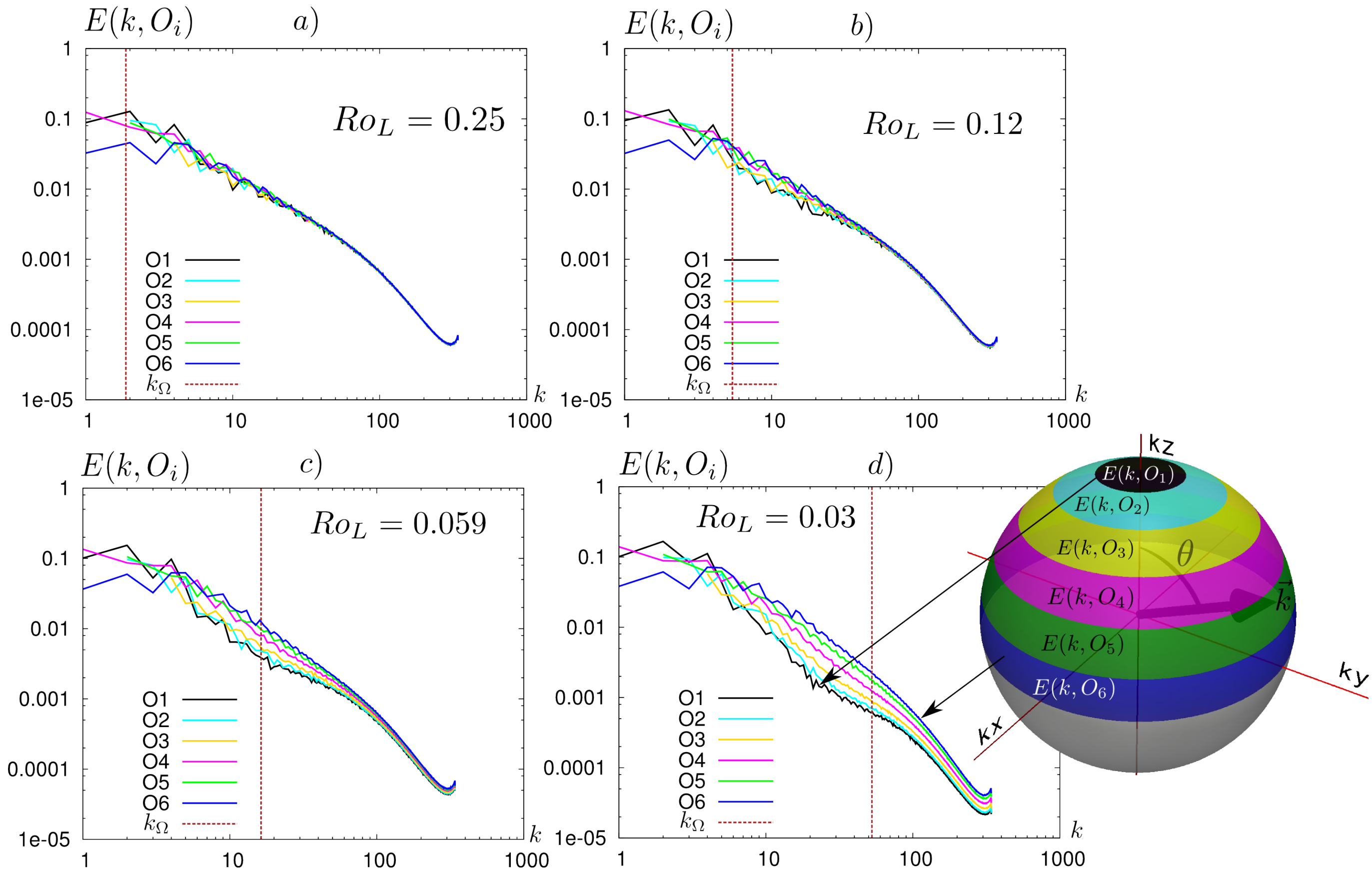

Figure 1. Angular power spectrum for different Rossby numbers (different rotating rates) and sketch of the angular decomposition by six spectral rings.

References

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