DEFORMATION OF TETRAHEDRA IN TURBULENCE

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Abstract The evolution of multi-particle structures provides insight into turbulent mixing and turbulence dynamics. In this talk, we discuss experimental and numerical findings of the deformation of tetrahedra in turbulence, both forward and backward in time. For short times, our theoretical analysis gives a result in close agreement with experimental and numerical data, and shows quantitatively the mechanism that breaks the symmetry in time. We also extend our analysis to turbulent flows with dilute polymer additives.

INTRODUCTION

It has long been known that in turbulent flows velocity gradients tend to flatten isotropic volumes [22, 1, 4] which is responsible for the enhancement of turbulent transport and mixing. To extend the analysis to the inertial range, over which the velocity field is not differentiable, a variety of models have been proposed to provide effective "coarse-graining" (see e.g., a recent review [13]). Among those models, the "tetrad approach" [5] is very promising as it requires only information from four fluid particles forming a tetrahedron, a minimum configuration required to describe a three-dimensional flow. An attractive feature of this approach is that the shape deformation of the tetrahedron is tightly related with the "perceived velocity gradient" of the turbulence at the scale of the tetrahedron [25, 17]. Experiments and numerical simulations show that, similar to previous results in the differentiable range, initially isotropic tetrahedra with their size in the inertial range of turbulent flows also deform into coplanar structures [18, 3, 12, 24, 8]. Quantitative understanding of this deformation process, to our knowledge, is still missing. Moreover, it has been shown that for the separation of two particles in turbulence, the separation rate when measured forward in time is different from that measured backward in time [19, 2]. Therefore it is natural to ask how would the shape of the tetrahedra deform in a frame moving backward in time.

Answers to these questions are not only directly related to turbulent mixing and transport, but also provide new insight to the understanding of inertial range dynamics of turbulence.

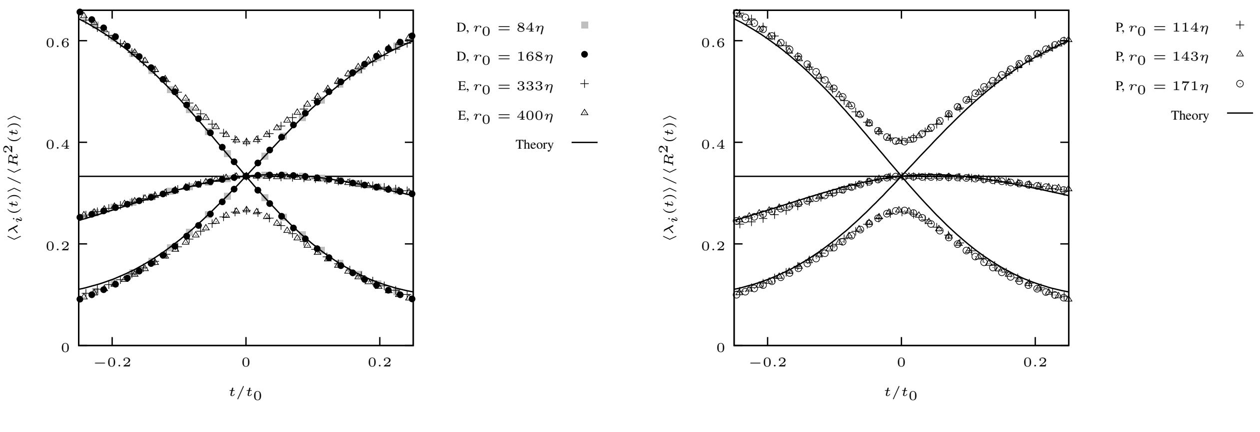

Figure 1. Time evolution of the averaged eigenvalues $\langle\lambda_a(t)\rangle$, normalized by the averaged radius of gyration $\langle R^2(t)\rangle$, for tetrahedra with different initial edge lengths r_0. Time has been normalized by $t_0 = (r_0^2/\epsilon)^{1/3}$, the Kolmogorov time corresponding to eddies of size r_0. *Left:* Experimental data (E), taken at R_λ=690 with a scale separation of L/η=2300, and DNS data (D), taken at R_λ=430 with L/η=480. *Right:* Experimental data with 10ppm polymer concentration (P), taken at $R_\lambda = 370$, L/η=936. The polymer energy dissipation rate ϵ_{pol} was derived from $< \delta\vec{u}\cdot\delta\vec{a} >_{t=0} = -2\epsilon_{pol}$ and differs from the pure water case.

DEFORMATION OF TETRAHEDRA IN NEWTONIAN FLUID TURBULENCE

We used data from Lagrangian Particle Tracking experiments to investigate the deformation of initially (nearly) isotropic tetrahedra in a von Kármán water flow between counter-rotating baffled disks at Taylor microscale Reynolds number $R_\lambda = 690$ [14]. The tetrahedra we analyze have an initial edge length r_0 well in the middle of the inertial range. We describe the shape of the tetrahedra by the three eigenvalues $\lambda_a(t)$, $a = 1, 2, 3$, of the shape tensor, which are intimately connected to the principal values of the moment of inertia of the tetrahedron [5, 18, 8]. With this definition,

$\sum_{a=1}^{3} \lambda_a(t) = R^2(t)$, where $R^2(t)$ is the radius of gyration of the tetrahedron. We further arrange the eigenvalues such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$. The case $\lambda_1 = \lambda_2 = \lambda_3$ represents an isotropic tetrahedron, $\lambda_1 \approx \lambda_2 > \lambda_3$ describes a flattened co-planar object (pancake-like shape), while $\lambda_1 > \lambda_2 \approx \lambda_3$ describes an elongated co-linear object (needle-like shape). We complement our experimental results with analysis of the DNS data at $R_\lambda = 430$ obtained from the JHU turbulence database [9, 26].

The evolution of $\lambda_a(t)/R^2(t)$ from both experiments and DNS are shown in Figure 1 (left), from which one can clearly see that the initially (nearly) isotropic tetrahedra deform into slightly elongated pancake-like structures at later times. For r_0 in the inertial range, the deformation process is self-similar if time is normalized by $t_0 = (r_0^2/\epsilon)^{1/3}$, the Kolmogorov time corresponding to eddies of size r_0. Furthermore, a clear asymmetry between the evolution forward and backward in time can be observed. The solid line shows our analytical result for the shape deformation, based on the tetrad approach. For short times, it is in close agreement with the experimental and numerical data and reveals the mechanism that results in the forward-backward asymmetry.

THE EFFECT OF POLYMER ADDITIVES

The addition of a small amount of flexible long-chain polymers into a fluid can drastically change its flow properties, see e.g. the elastic turbulence [7] and the drag reduction phenomena [21]. The polymer-turbulence interaction has been studied intensively [11, 20, 16] and the effects of polymers on different scales and on the energy cascade have been observed [10, 6, 15]. We report here the deformation of tetrahedra in a turbulent flow of dilute polymer solutions as an attempt to investigate the influence of polymers on turbulent mixing. Details on the experimental setup and the polymer data can be found in [23]. Comparison with the analytical result for Newtonian flows (see Figure 1, right) shows that the effect of polymers on the turbulence energy cascade need to be taken into account when analyzing the shape deformation.

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