PRANDTL NUMBER DEPENDENCE OF STATISTICS IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Abstract We study the dependence of statistical properties on the Prandtl number in turbulent Rayleigh-Bénard convection in a cylindrical cell using three-dimensional direct numerical simulations. The Prandtl number is varied between $Pr = 0.07$ and 7 for a fixed Rayleigh number of $Ra = 3 \times 10^9$. It is found that the dynamics of the large-scale circulation is altered by the Prandtl number variation. In contrast, the probability density function of small-scale quantities such as the local heat currents are less affected by this parameter change. Implications for the global heat transfer are discussed.

INTRODUCTION

One of the most comprehensively studied turbulent flows is the Rayleigh-Bénard (RB) convection, in which a complex three-dimensional turbulent motion is initiated by heating the fluid from below and cooling from above. Convection appears in many flows in nature and technology (see e.g. Ref. [1] for a recent review). They cover a wide range of different Prandtl numbers Pr, one of the three control parameters beside the Rayleigh number Ra and the aspect ratio $\Gamma = D/H$, where D is the diameter and H the height of the cell. The system response to this set of control parameters are a turbulent heat transport measured by the Nusselt number Nu and a momentum transport measured by the Reynolds number Re.

Systematic studies of the Pr dependence at higher Rayleigh numbers are difficult in laboratory experiments and have been done therefore mostly in numerical investigations (see e.g. Refs. [3, 4]). Direct numerical simulations (DNS) of RB convection at both, high and low Prandtl numbers, are however challenging in terms of grid resolution since the thicknesses of the boundary layers of temperature and velocity fields differ strongly when $Pr \ll 1$ or $Pr \gg 1$. How this circumstance affects their coupling and the boundary layer dynamics as a whole is yet poorly understood. This sets the stage for our present effort. To give an example of the numerical efforts that have to be taken: in our studies, it is found that decreasing Pr by an order of magnitude has roughly the same effect as increasing Ra by an order of magnitude in terms of the required grid resolution.

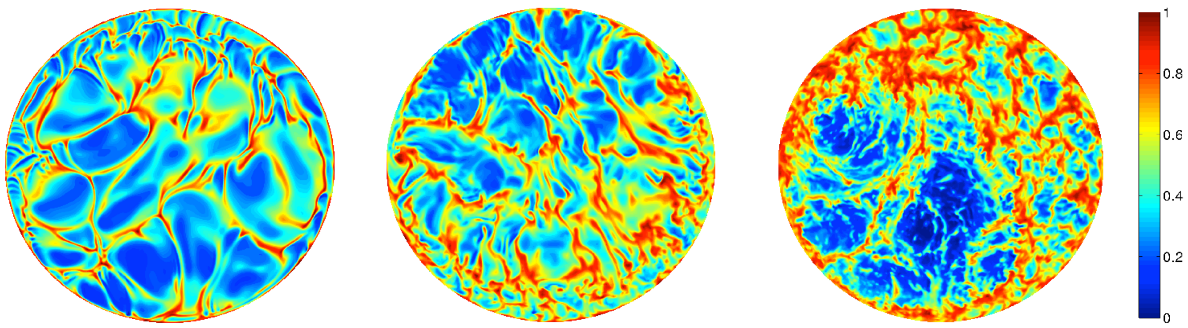

Figure 1. Temperature contours of an instantaneous snapshot plotted in the boundary layer plane. Left panel: $Pr = 7.0$. Mid panel: $Pr = 0.7$. Right panel: $Pr = 0.07$.

DIRECT NUMERICAL SIMULATIONS FOR DIFFERENT PRANDTL NUMBERS

We solve the three-dimensional Boussinesq equations numerically. The height of the cell H, the free-fall velocity $U_f = \sqrt{g\alpha\Delta T H}$ and the imposed temperature difference ΔT are used to rescale the equations of motion. This results in the following dimensionless form of the equations of motion

$$\nabla \cdot \mathbf{u} = 0\,, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \sqrt{\frac{Pr}{Ra}}\nabla^2 \mathbf{u} + T\mathbf{e}_z\,, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T = \frac{1}{\sqrt{RaPr}}\nabla^2 T\,, \tag{3}$$

where the two control parameters are defined as

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa}\,, \qquad Pr = \frac{\nu}{\kappa}\,. \tag{4}$$

They contain gravity acceleration g, thermal expansion coefficient α, kinematic viscosity ν, and thermal diffusivity κ beside quantities which have been defined above. Throughout the study, we set $\Gamma = 1$. At all walls no-slip boundary conditions for the fluid are applied, $\mathbf{u} = 0$. The side walls are adiabatic, i.e., the normal derivative of the temperature field vanishes, $\partial T/\partial \mathbf{n} = 0$. The top and bottom plates are held at a fixed temperatures $T = 0$ and 1. The presented simulations are done for Prandtl numbers $Pr = 0.07, 0.7$ and 7 and for fixed Rayleigh number $Ra = 3 \times 10^9$. The equations are discretized in cylindrical coordinates with a second-order finite difference scheme [5, 6]. The time advancement is done by a third-order Runge-Kutta scheme. Contours of the temperature field for the three different Pr cases are shown in Fig. 1. It is observed that at higher Pr (left panel), the sheet-like plumes are most pronounced compared to lower Pr cases.

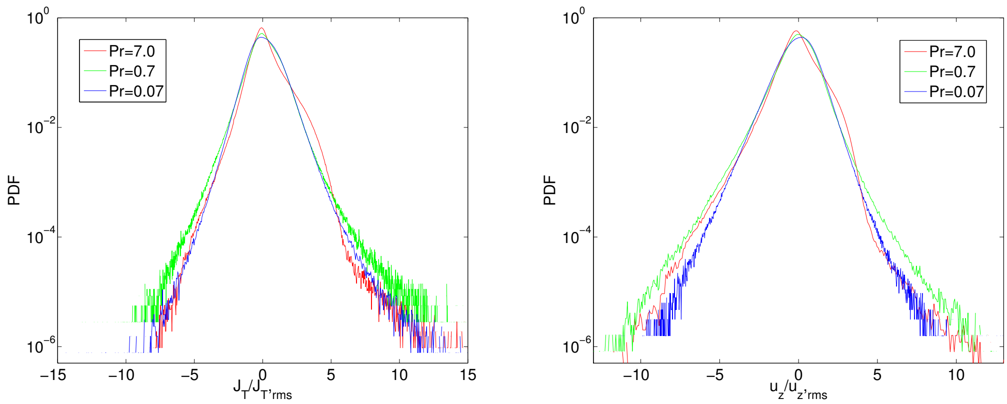

Figure 2. Left panel: PDFs of the local heat current J_T at the boundary layer plane for different Pr as indicated in the legend. Right: PDFs of the vertical velocity u_z at the boundary layer plane for different Pr. Both quantities are normalized by the corresponding root mean square value.

Similar to previous studies at $Pr = 0.7$ [2], we analyzed the large-scale circulation which builds up in a convection cell. It is found the angular variations of the wind direction are smaller in amplitude when the Prandtl number is enhanced or decreased. The mean amplitude of the large-scale wind speed decreases with increasing Prandtl number. The mean profiles of the thermal and velocity boundary layer agree better with the predictions from the classical boundary layer theory when the so-called dynamical rescaling is applied (see e.g. Ref. [2] for more details). A further example of our analysis is reported in Fig. 2. In this figure the probability density functions (PDFs) of the local heat current $J_T(\mathbf{x}, t) = u_z T - \kappa \partial T/\partial z$ and the vertical velocity u_z are compared for three different Prandtl numbers in the boundary layers. The PDFs are highly intermittent in all cases which is indicated by the extended tails. We found however that the variations with the Prandtl remain small. This might be a fingerprint of the small modification of the global heat transfer as a function of the Prandtl number.

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