STEWARTSON LAYER INSTABILITY IN THE PROBLEM OF THE VIBRATIONAL HYDRODYNAMIC TOP

Viktor Kozlov, Nikolai Kozlov & Stanislav Subbotin
General and experimental physics department, Perm State Humanitarian-Pedagogical University, Perm, Russia

Abstract The flows are studied experimentally, which are excited at differential rotation of a free light spherical body centrifuged in a cavity rotating around horizontal axis and filled with viscous liquid. The differential rotation is excited by an external force field rotating in the cavity frame (vibrational hydrodynamic top [1]). At weak differential rotation the flow has the shape of the Taylor–Proudman column of a circular cross-section. The occurrence of the column instability is found at increase of the differential rotation speed of the sphere, when a non-axisymmetric azimuthal flow develops. The comparison is done of the stability thresholds of the axisymmetric flow with the classic case, when the sphere rotation axis is fixed on the cavity rotation axis. The instability development thresholds are different by one order of magnitude. The conclusion is done that in case of the vibrational hydrodynamic top a new type of the Stewartson layer instability is found.

DIFFERENTIAL ROTATION

A light spherical body of the radius r is placed in a cylindrical cavity, filled with viscous incompressible liquid. The cavity is set at rotation around a horizontal axis, sufficiently fast and steady, so that under the action of the centrifugal force the light sphere comes from the side (cylindrical) wall to the axis. The cavity rotation speed Ω_{rot} is always higher than the sphere rotation speed Ω_s. This is due to the gravity action: in the cavity frame of reference the vector **g** rotates opposite to the cavity rotation, inducing the circular oscillations of the sphere in the plane perpendicular to the rotation axis. Such forced oscillations lead to the averaged mass force generation in the Stokes boundary layer in the proximity of the body surface. As a result, the retrograde differential azimuthal rotation of the body relative to the cavity is excited [1]. The differential rotation speed $\Delta\Omega \equiv \Omega_s - \Omega_{rot}$ is determined by the ratio of the gravity force to the centrifugal force $\Gamma \equiv g / \Omega_{rot}^2 r$, which is varied in the experiment via the parameter Ω_{rot}. At slow differential rotation the flow has a shape of the Taylor–Proudman column with the circular cross-section (fig. 1, *a*). The column is extended along the cavity rotation axis, its transverse dimension coinciding with the sphere diameter. The cylindrical column boundary is formed by the Stewartson shear layer, which compensates the discontinuity of the flow velocity azimuthal component.

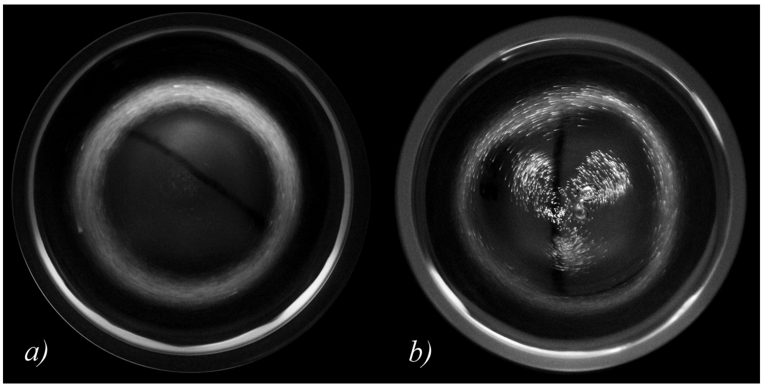

Figure 1. Photographs of the Taylor–Proudman column in the cross-section. In the centre there is the light sphere.

STABILITY OF THE AXISYMMETRIC FLOW

At increase of the differential rotation speed the shear layer goes through an instability, which is manifested in the appearance of a two-dimensional azimuthal wave propagating on the column surface [2]. It is found that inside the column the flow is not solid-state. Simultaneously with the wave on the outer boundary, inside the column a system of several vortices rotates synchronously with it. On fig. 1, *b*, the azimuthal wave number m and the number of vortices are equal to 3. The stability of the axisymmetric flow is determined by two dimensionless parameters: Rossby number $Ro \equiv \Delta\Omega/\Omega_{rot}$ and dimenionless frequency $\omega \equiv r^2\Omega_{rot}/\nu$ (ν – kinematic viscosity). With the increase of ω the critical Rossby number decreases according to the law $|Ro_C| \sim \omega^{-1.0}$. For comparison with the classic type of the shear layer instability, additional experiments were carried out, where the sphere was fixed on a thin rod on the cavity rotation axis.

In this case the differential rotation was imposed by using two motors separately for the cavity and the body. The results of both experimental series are plotted on fig. 2. The critical Rossby number dependence on the dimensionless frequency in both cases is similar: the azimuthal wave number *m* increases with ω and decreases with Ro, the exponents are approximately equal. However the thresholds of the instability onset are different by one order of magnitude. The results with the fixed sphere are in agreement with the experiments [3,4]. The second essential difference between two cases is the perturbation propagation phase velocity. In the experiments with the free sphere the wave phase velocity is inferior to the velocities of both cavity and sphere (in the laboratory frame). When the sphere rotation axis is fixed, the wave velocity takes a value intermediate between the velocities of the sphere and the cavity.

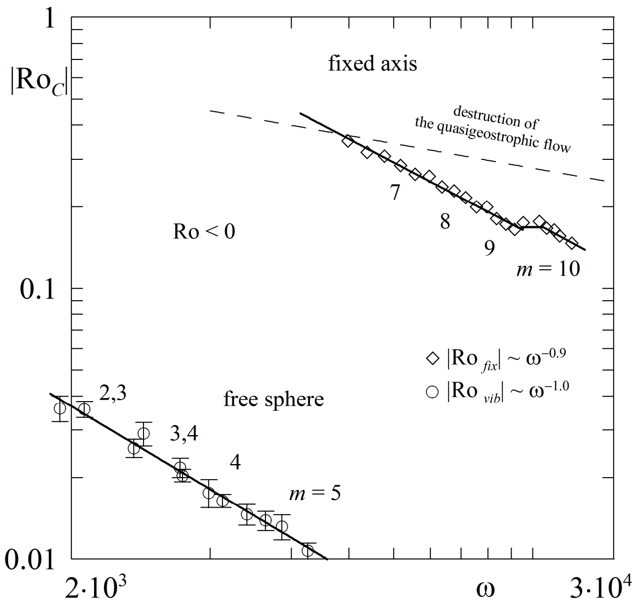

Figure 2. Comparison of the stability threshold of the axisymmetric flow excited by the free sphere and the sphere fixed on the cavity rotation axis.

Thus, it is experimentally shown that the formation of the Stewartson shear layer is possible in case of differential rotation of a free light body excited by an external oscillating force. However the dynamics of such layer is qualitatively different from what is observed for the classic Stewartson layer. The crucial differences are lowering of stability of the shear flow, change of the phase velocity of the wave propagation, development of the vortex structures inside the liquid column. This allows considering a new type of instability of the Stewartson layer, manifesting itself at differential rotation of the vibrational hydrodynamic top.

Acknowledgements

The work is done in the frame of the strategic development program of PSHPU (project 040-M) and the task of the Ministry of education and science of Russia N 1.2783.2011.

References

[1] V.G. Kozlov, N.V. Kozlov. Vibrational hydrodynamic gyroscope. *Dokl. Ross. Akad. Nauk* **415**: 759–762, 2007.

[2] A.A. Ivanova, N.V. Kozlov, S.V. Subbotin. Vibrational dynamics of a light spherical body in a rotating cylinder filled with a fluid. *Fluid Dynamics* **47**: 683–693, 2012.

[3] R. Hide, C.W. Titman. Detached shear layers in a rotating fluid. *J. Fluid Mech* **29**: 39–60, 1967.

[4] R. Hollerbach, B. Futterer, T. More, C. Egbers. Instabilities of the Stewartson layer. Part 2. Supercritical mode transitions. *Theoret. Comput. Fluid Dynamics* **18**: 197–204, 2004.