MATHEMATICAL ANALYSIS OF HEAT TRANSPORT IN TURBULENT CONVECTION

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Abstract We review the current state of mathematical analysis for bounds on heat transport in Bénard convection and compare the most recent rigorous results for several scenarios with predictions of heuristic theoretical arguments and/or direct numerical simulations.

The Boussinesq approximation to the Navier-Stokes equations are, in a standard non dimensional form,

$$\begin{aligned} \frac{1}{Pr}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) + \nabla p &= \nabla^2\mathbf{u} + Ra\mathbf{k}T \\ \nabla\cdot\mathbf{u} &= 0 \\ \frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T &= \nabla^2 T. \end{aligned}$$

where $\mathbf{u}$ is the fluid's velocity vector field, T is the temperature field, $\mathbf{k}$ is the vertical unit vector directing the buoyancy force, and Pr and Ra are, respectively, the Prandtl and Rayleigh numbers. This system of partial differential equations is used to model a wide variety of Bénard convection problems where a layer of fluid is heated from below, but it is not fully defined until appropriate boundary conditions are specified. For example the fluid may satisfy no-slip or free-slip (no-stress) velocity boundary conditions, and either the temperature or the heat flux may be set at the boundaries. In the case of pure Bénard-Marangoni convection there is no buoyancy force: then the temperature and flow are coupled by thermal variations setting stresses on a surface. These different situations determine distinct relations between the heat transport and the temperature gradient across the layer. In this presentation we describe some recent rigorous results relating these quantities for a selection of physical problems of current interest [1, 2, 3, 4].

References

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