VORTEX STRUCTURES OF 3D SEPARATED FLOWS OF STRATIFIED VISCOUS FLUID

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Abstract Direct numerical simulation of the stratified viscous fluid flows around a sphere and a 2D square cylinder on supercomputers and the modern visualization techniques gives us opportunity to better understand the complex transformations of the vortex structures of the wake with changing of the main non-dimensional parameters: Reynolds and internal Froude numbers.

Unsteady 3D separated and undulatory fluid flows around a horizontally moving bodies are very wide spread phenomena in the nature. Mathematical modeling of such flows on supercomputers give us opportunity to better understand the complex transformations of the 3D vortex structures of wake with changing of the main non-dimensional parameters (Reynolds (Re) and internal Froude (Fr) numbers). The existing experimental data give us opportunity to confirm the results of our modeling at some values of Re and Fr.

NUMERICAL METHOD SMIF AND THE λ_2- AND β-VISUALIZATION TECHNIQUES

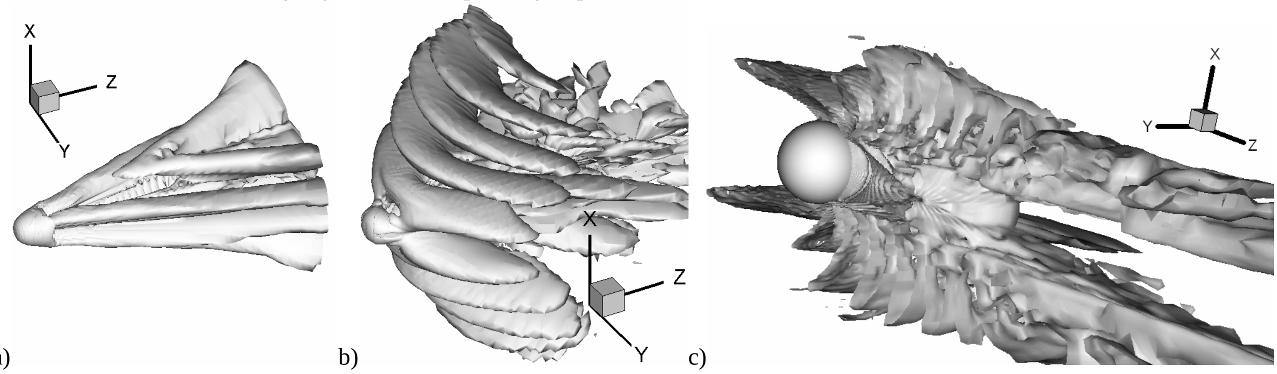
The density stratified viscous fluid flows have been simulated on the basis of the Navier-Stokes equations in the Boussinesq approximation (including the diffusion equation for the stratified component (salt)) with four dimensionless parameters: $Fr = U/(N{\cdot}d)$, $Re = U{\cdot}d/\nu$, $C = \Lambda/d \gg 1$, $Sc = \nu/\kappa = 709.22$, where U is the scalar of the body velocity, d is the diameter of body; Λ is the buoyancy scale, which is related to the buoyancy frequency N and period T_b ($N = 2\pi/T_b$, $N^2 = g/\Lambda$); g is the scalar of the gravitational acceleration; ν is the kinematical viscosity, κ is the salt diffusion coefficient. The density $\rho = \rho_0(1 - x/(2C) + S)$ where x is a vertical Cartesian coordinate, S is a dimensionless perturbation of salinity.

For solving of the Navier-Stokes equations the Splitting on physical factors Method for Incompressible Fluid flows (SMIF) with hybrid explicit finite difference scheme (second-order accuracy in space, minimum scheme viscosity and dispersion, capable for work in wide range of *Re* and *Fr* and monotonous) has been developed and successfully applied [1-2].

For the visualization of the 3D vortex structures in the wake the isosurfaces of β and λ_2 have been drawing, where β is *the imaginary part of the complex-conjugate eigen-values of the velocity gradient tensor* **G** [3] (fig. 1), λ_2 is *the second eigen-value of the* $\mathbf{S}^2 + \mathbf{\Omega}^2$ *tensor*, where **S** and **Ω** are the symmetric and antisymmetric parts of **G** [4] (fig. 2a). The good efficiency of this β-visualization technique has been demonstrated in [5].

RESULTS

The following classification of the viscous fluid flow regimes around a sphere is given in [6]: 1) $200 < Re \le 270$ – a steady double-thread wake; 2) $270 < Re < 300$ - a double-thread with waves; 3) $300 < Re < 420$ – a procession of the vortex loops (facing upwards); 4) $420 < Re < 800$ – a procession of the vortex loops with the rotation of the shear layer; 5) $800 < Re < 3.7{\cdot}10^5$ - a procession of the vortex loops with the shear layer instability; 6) $Re > 3.7{\cdot}10^5$ - the turbulent boundary layer. *Owing to our investigations* the detailed formation mechanisms of vortices (FMV) in the sphere wake have been described for $200 \le Re \le 1000$ [5]. In particular it was shown that the detailed FMV for $270 < Re \le 290$, $290 < Re \le 320$ and $320 < Re \le 400$ are different. At $5{\cdot}10^4 \le Re \le 4{\cdot}10^5$ the monotonous reduction of the time-averaged total drag coefficient has been observed (from value 0.455 to 0.155) due to the laminar-turbulent transition in the boundary layer [7]. It was shown that this drag crisis manifests itself to us through the formation of the separated bubbles within the boundary layer (near the primary separation line).



a) b) c)

Figure 1. Vortex structures of the sphere wake at $Re = 100$: a-c – $Fr = 2$; 0.5; 0.08; a-c – $\beta = 0.005$; 0.02; 0.005.

The following classification of stratified viscous fluid flow regimes around a sphere at $Re < 500$ [7-8] has been obtained by SMIF (fig. 1-2): **I**) $Fr > 10$ – *the homogeneous case*; **II**) $1.5 \le Fr \le 10$ – *the quasi-homogeneous case* (with four additional threads connected with the vortex sheet (or a shear layer) surrounding the sphere, fig. 1a, 2b); **III**) $0.9 < Fr < 1.5$ - *the non-axisymmetric attached vortex* in the recirculation zone (RZ) (fig. 2c); **IV**) $0.6 < Fr \le 0.9$ – *the two symmetric vortex loops in* RZ (fig. 2d); **V**) $0.4 \le Fr \le 0.6$ – *the absence* of RZ (fig. 1b, 2d); **VI**) $0.25 < Fr < 0.4$ – *a new RZ* (fig. 2e); **VII**) $Fr \le 0.25$ - *the two vertical vortices* in new RZ (bounded by internal waves (IWs)) (fig. 1c). At $Fr \le 0.3$, $Re > 120$ a periodical generation of the vortex loops (facing left or right) has been observed. The corresponding Strouhal numbers $0.19 < St = f \cdot d/U < 0.24$ (where f is the frequency of shedding) and horizontal and vertical separation angles are in a good agreement with the experiments [9-10]. The drag coefficients also correspond to experimental values. The interesting transformation of the four main threads (at $Fr = 2$) into *the high gradient sheets of density* near the sphere poles (before the sphere) (at $Fr = 0.08$) is shown at fig. 1.

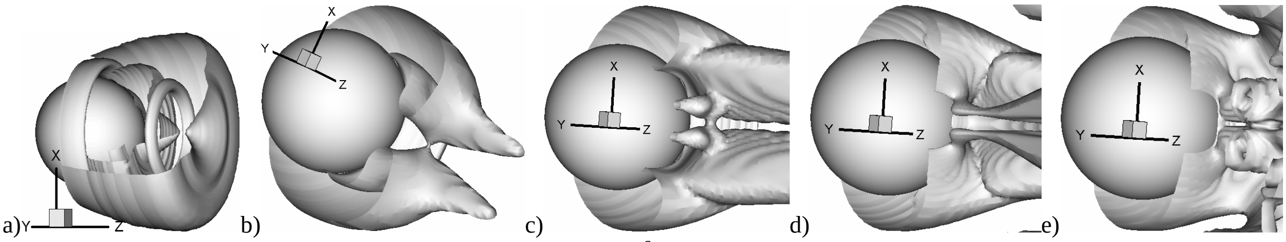


Figure 2. The isosurfaces of $\lambda_2 = -10^{-6}$ & -0.16 at $Re = 200$ ($Fr = \infty$) (a) and the isosurfaces of β at $Re = 100$: b-e – Fr =2, 1, 0.6, 0.35; b-e – β = 0.15, 0.1, 0.087, 0.087.

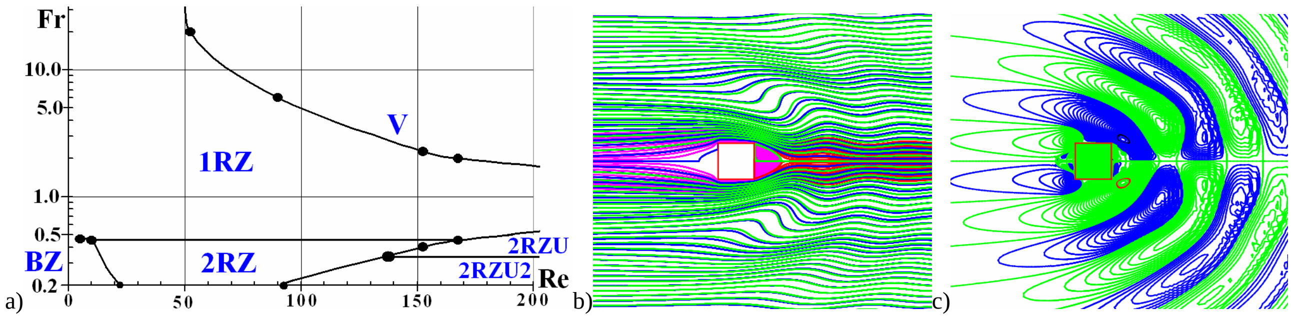


Figure 3. The classification of the regimes of 2D stratified viscous fluid flows around a square cylinder (a), stream lines (b) and isolines of $S_z = \partial\rho/\partial z$ (c) ($\delta S_z = 0.227$) around a square cylinder at $Fr = 0.3$, $Re = 50$; $t = 332.07 \cdot T_b$.

The following classification of stratified viscous fluid flow regimes around a 2D square cylinder has been obtained by SMIF [1-2] at $Re < 200$ (fig. 3a): **HC**) $Fr > 20$ – a steady symmetrical RZ with length L_0 (*the homogeneous case*); **1RZ**) a steady symmetrical RZ with length $L < L_0$ and IWs; **V**) a procession of the vortices in the wake; **2RZ**) two steady symmetrical RZs before and after the square cylinder (fig. 3b-c); **BZ**) a steady symmetrical RZ (or blocking zone (BZ) with the length L_b) before the square cylinder, IWs and *the high gradient sheet of density* are forming on the horizontal axis z after the cylinder (L_b is rapidly increasing with decreasing of Fr); **2RZU**) a procession of the vortices (bounded by IWs); **2RZU2**) an unsteadiness inside RZ with steady symmetric boundaries and the symmetric IWs.

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