GLOBAL BIFURCATIONS TO SUBCRITICAL TURBULENT MAGNETOROTATIONAL DYNAMO ACTION IN KEPLERIAN SHEAR FLOW

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Abstract Magnetorotational (MRI) dynamo action in Keplerian shear flow is a three-dimensional, nonlinear magnetohydrodynamic mechanism whose study is relevant to dynamo theory and to the understanding of accretion processes in astrophysics. Transition to this form of dynamo action is subcritical and shares many of the characteristics of subcritical transition to turbulence in hydrodynamic shear flows. This suggests that both types of flows become active through similar generic bifurcation mechanisms, which in both cases have eluded detailed understanding so far. We build on recent work on both types of problems to investigate numerically the bifurcation mechanisms at work in the MRI dynamo problem. The emergence of three-dimensional chaos and transient magnetohydrodynamic turbulence in this problem is shown to be primarily associated with global homoclinic and heteroclinic bifurcations involving the stable and unstable manifolds of nonlinear MRI dynamo cycles born out of saddle node bifurcations. The detailed results strongly suggest that nonlinear MRI dynamo cycles are key actors of the transition in this system. This opens new perspectives to assess the conditions of excitation of instability-driven dynamos in Nature and in laboratory experiments.

ASTROPHYSICAL AND PHYSICAL CONTEXT

Instability-driven dynamos, i.e. dynamos relying partly on the development of magnetohydrodynamic instabilities such as the magnetorotational instability (MRI) in Keplerian flow, are very interesting candidates to explain sustained magnetic activity and turbulence in a variety of differentially rotating astrophysical bodies such as stellar interiors or accretion disks [1, 5, 16, 2, 11] and could perhaps be observed in dynamo experiments in the near future. Our understanding of the transition to this form of dynamo action remains very crude though. It is known to require finite amplitude perturbations and to be essentially non-kinematic, i.e. the excitation of the magnetic field is completely coupled to the excitation of velocity fluctuations. Recent results on the MRI dynamo [11, 12, 10, 6] strongly suggest that this particular dynamo transition has much in common with the transition to hydrodynamic turbulence in linearly stable shear flows [3].

MAIN RESULTS

In this communication, we present numerical results that reveal the fascinating complexity of the transition to turbulent MRI dynamo action in Keplerian shear flow, as studied in the so-called incompressible shearing box framework. The detailed results are presented in [6, 13]. The first result is that the system first exhibits features typical of a chaotic repeller such as fractal patterns in maps of turbulence lifetime as the magnetic Reynolds number *Rm* is increased (Fig. 1).

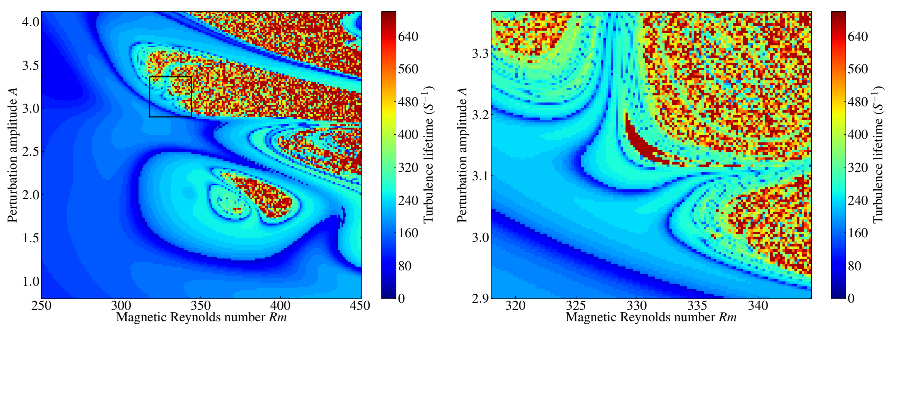

Figure 1. Maps of turbulence lifetime as a function of the amplitude of the initial condition A and *Rm* for a given, randomly generated spatial form of initial condition. The rightmost figure is a high-resolution map ($\delta Rm = 0.2, \delta A = 0.004$) computed for a restricted parameter range (indicated by the black rectangle) of a lower resolution map on the left [13].

We then investigate what causes this kind of transition pattern. The dynamics in the transitional regime is found to be strongly structured around unstable three-dimensional nonlinear MRI dynamo cycles (periodic orbits) which can be computed with precision using Newton's method. One such cyclic MRI dynamo state is depicted in Fig. 2. These cycles are large-scale coherent structures that cannot be described simply in terms of standard mean-field dynamo theory.

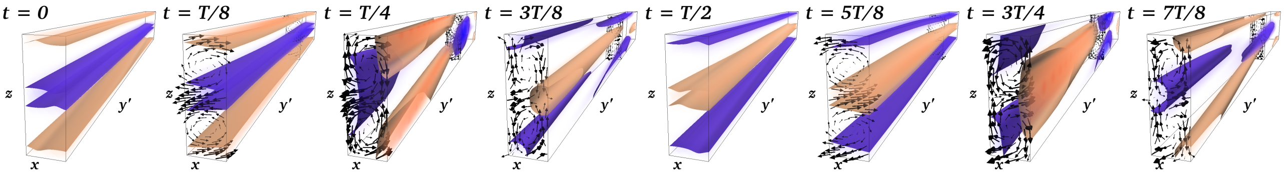

Figure 2. Isosurfaces of magnetic field $B = 0.9$ colored by toroidal field B_y (in standard dynamo terminology) at various stages of a MRI dynamo cycle of period T (positive B_y in red/light gray, negative B_y in blue-violet/dark gray). The arrows field in the poloidal plane $y = 0$ plane traces non-axisymmetric poloidal MRI velocity perturbations [6].

They appear in pairs at saddle-node bifurcations as *Rm* is increased. The main result of the present work is that these simple bifurcations are very quickly followed by global heteroclinic and homoclinic bifurcations whose outcome is the formation of Poincaré tangles [9] involving the stable and unstable manifolds of cycles (Fig. 3). As expected from Smale's theorem [15], these bifurcations result in a transition of the whole system to transient chaotic states.

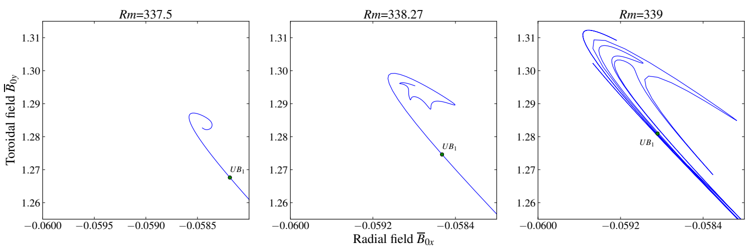

Figure 3. 2D projection of the unstable manifold of an upper-branch cycle (UB_1) born out of a saddle-node bifurcation at *Rm* $= 327.4$, as a function of *Rm*. The projection is in the plane of radial ($\overline{B}_{0x}$) vs. toroidal ($\overline{B}_{0y}$) axisymmetric field. At *Rm* $= 339$, the manifold has a folded and stretched geometry typical of a homoclinic tangle [13].

CONCLUSIONS

These results are reminiscent of several studies on the transition to turbulence of hydrodynamic shear flows [14, 8, 17, 7]. They strongly suggest that nonlinear cycles are key actors of the transition in these different systems and, in the present problem, provide the pathway to large-scale injection of energy into MHD turbulence. This improved understanding of the role of individual cycles in the global MRI dynamo transition process opens new perspectives to assess the conditions of excitation of instability-driven dynamos in Nature, such as the much debated dependence of the MRI dynamo on the magnetic Prandtl number [4], and in laboratory dynamo experiments involving shear flows.

References

[1] A. Brandenburg, A. Nordlund, R. F. Stein, and U. Torkelsson. Dynamo-generated turbulence and large-scale magnetic fields in a Keplerian shear flow. *ApJ*, **446**:741, 1995.
[2] K. S. Cline, N. H. Brummell, and F. Cattaneo. Dynamo action driven by shear and magnetic buoyancy. *ApJ*, **599**:1449, 2003.
[3] B. Eckhardt, T. M. Schneider, B. Hof, and J. Westerweel. Turbulence transition in pipe flow. *ARFM*, **39**:447, 2007.
[4] S. Fromang, J. Papaloizou, G. Lesur, and T. Heinemann. MHD simulations of the magnetorotational instability in a shearing box with zero net flux. II. The effect of transport coefficients. *A&A*, **476**:1123, 2007.
[5] J. F. Hawley, C. F. Gammie, and S. A. Balbus. Local three-dimensional simulations of an accretion disk hydromagnetic dynamo. *ApJ*, **464**:690, 1996.
[6] J. Herault, F. Rincon, C. Cossu, G. Lesur, G. I. Ogilvie, and P.-Y. Longaretti. Periodic magnetorotational dynamo action as a prototype of nonlinear magnetic-field generation in shear flows. *PRE*, **84**:036321, 2011.
[7] T. Kreilos and B. Eckhardt. Periodic orbits in plane couette flow. *Submitted*, 2012.
[8] J. Moehlis, B. Eckhardt, and H. Faisst. Fractal lifetimes in the transition to turbulence. *Chaos*, **14**:11, 2004.
[9] J. Palis and F. Takens. *Hyperbolicity and sensitive chaotic dynamics at homoclinic bifurcations*. Cambridge University Press, 1993.
[10] E. Rempel, G. Lesur, and M. R. E. Proctor. Supertransient magnetohydrodynamic turbulence in Keplerian shear flows. *PRL*, **105**:044501, 2010.
[11] F. Rincon, G. I. Ogilvie, and M. R. E. Proctor. Self-sustaining nonlinear dynamo process in Keplerian shear flows. *PRL*, **98**:254502, 2007.
[12] F. Rincon, G. I. Ogilvie, M. R. E. Proctor, and C. Cossu. Subcritical dynamos in shear flows. *Astronomische Nachrichten*, **329**:750, 2008.
[13] A. Riols, F. Rincon, C. Cossu, G. Lesur, P.-Y. Longaretti, G. I. Ogilvie, and J. Herault. Global bifurcations to subcritical turbulent magnetorotational dynamo action in Keplerian shear flow. *submitted to JFM*, 2013.
[14] A. Schmiegel. Fractal stability border in plane Couette flow. *PRL*, **79**:5250, 1997.
[15] S. Smale. Differentiable dynamical systems. *Bull. Amer. Math. Soc.*, **73**:747, 1967.
[16] H. C. Spruit. Dynamo action by differential rotation in a stably stratified stellar interior. *A&A*, **381**:923, 2002.
[17] L. van Veen and G. Kawahara. Homoclinic tangle on the edge of shear turbulence. *PRL*, **107**:114501, 2011.