ENERGY TRANSFERS IN A FORCED HOMOGENEOUS TURBULENCE EXPERIMENT UNDER ROTATION

Campagne Antoine[1], Gallet Basile[1], Billant Paul[2], Moisy Frédéric[1] & Cortet Pierre-Philippe[1]

[1]*Laboratoire FAST, CNRS, Univ Paris-Sud, UPMC Univ Paris 06, France*

[2] *LadHyX, CNRS, Ecole Polytechnique, Palaiseau, France*

Abstract We investigate the interplay between the cyclone-anticyclone asymmetry and the anisotropic energy transfers in a statistically stationary, homogeneous and axisymmetric turbulence experiment in a rotating frame. Turbulence is forced by a new setup made of 10 vortex dipole generators arranged in arena in a large water tank mounted on a rotating platform. Energy transfers are characterized by the energy flux density $\mathbf{F}(\mathbf{r}) = \langle \delta\mathbf{u}\,(\delta\mathbf{u})^2\rangle$, computed from velocity increments $\delta\mathbf{u}$ over separation vector $\mathbf{r}$ measured by corotating particle image velocimetry. Interestingly, because of the cyclone-anticyclone asymmetry present at Rossby number of order 1, the mirror symmetry with respect to any vertical plane is broken, resulting in a nearly azimuthal energy flux density $\mathbf{F}(\mathbf{r})$. The influence of this additional degree of freedom on the dynamics of the energy transfers is discussed.

INTRODUCTION

The statistics of the energy transfers between spatial scales is often considered as the cornerstone of turbulence theory [1, 2]. Energy transfers can be described for homogeneous turbulence thanks to the Kármán-Howarth-Monin (KHM) equation [1, 3],

$$\partial_t R/2 = \Pi(\mathbf{r}, t) + \nu\nabla^2 R + \phi(\mathbf{r}, t), \tag{1}$$

where $R(\mathbf{r}, t) = \langle \mathbf{u}(\mathbf{x}, t) \cdot \mathbf{u}(\mathbf{x} + \mathbf{r}, t)\rangle$ is the two-point velocity correlation, $\Pi(\mathbf{r}, t) = \nabla \cdot \mathbf{F}/4$ the energy flux, $\phi(\mathbf{r}, t)$ the energy injection term, and $\langle\cdot\rangle$ stands for spatial and ensemble averages. The energy flux is defined from the vector flux density $\mathbf{F}(\mathbf{r}) = \langle \delta\mathbf{u}\,(\delta\mathbf{u})^2\rangle$, where $\delta\mathbf{u} = \mathbf{u}(\mathbf{x} + \mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t)$ is the velocity increment over the separation vector $\mathbf{r}$. For 3D stationary turbulence at large Reynolds number, Eq. (1) yields in the inertial range an energy flux conservation $\Pi(\mathbf{r}) = -\epsilon$, where $\epsilon = -\nu\nabla^2 R(0)$ is the dissipation rate [3, 4, 5]. If isotropy is further assumed, this is equivalent to the famous Kolmogorov 4/5th law, describing the usual energy cascade from large to small scales.

Remarkably, the flux conservation $\Pi(\mathbf{r}) = -\epsilon$ holds even for anisotropic turbulence, including rotating turbulence [5, 6]. In this case, the energy transfers become anisotropic, with favored transfers towards 2D horizontal modes [5, 6, 7] (the rotation axis is vertical by convention), raising, as in strictly 2D turbulence, the possibility of an inverse cascade. In 2D turbulence (with or without rotation), the additional constraint of enstrophy conservation indeed yields an inverse energy cascade [4], characterized by $\Pi(\mathbf{r}, t) = +P$ in an "upper" inertial range (for scales larger than the injection scale), where P is the rate of energy injection. Clues on inverse cascade in 3D turbulence experiments under rotation $\boldsymbol{\Omega}$ have been reported through a change of sign of the third order moment of longitudinal velocity increments [8, 9], although its connection with a genuine 2D inverse cascade remains debated.

Another important feature of rotating turbulence is the symmetry breaking between cyclonic and anticyclonic vorticity which occurs for Rossby number of order 1 (i.e., for $\langle\omega_z^2\rangle^{1/2} \simeq \Omega$, where $\omega_z = \boldsymbol{\omega} \cdot \boldsymbol{\Omega}/\Omega$ is the vertical vorticity) [9, 10]. This non-linear 3D effect is expected even for purely 2D forcing [11, 12], indicating the inevitable influence of 3D turbulent motions in rotating turbulence at finite Rossby number. As a result, homogeneous axisymmetric rotating turbulence lacks mirror symmetry with respect to vertical planes. Although this *weaker* axisymmetry property has no direct influence on scalar quantities such as $R(\mathbf{r}, t)$ and $\Pi(\mathbf{r}, t)$, it provides an additional degree of freedom for the flux density $\mathbf{F}(\mathbf{r}, t)$. Characterizing the geometrical properties of $\mathbf{F}(\mathbf{r}, t)$ as a function of the Reynolds and Rossby numbers is a fundamental issue for rotating turbulence, which requires a well controlled experiment (in order to build a homogeneous turbulence) and anisotropic velocity measurements.

EXPERIMENTAL SETUP

The experiments are performed in a water-filled tank of 1.25×1.25 m^2 base and 0.6 m depth mounted on a precision rotating platform of 2 m diameter. The angular velocity Ω of the platform is set in the range $\Omega = 2$ to 16 rpm, with relative fluctuations less than 10^{-3}. The flow is excited by 10 vortex dipole generators organized in arena (Figs. 1a-b). A vortex dipole generator is composed of two symmetric vertical plates whose closing creates a pair of counter-rotating vertical columnar vortices (Fig. 1a) [11, 12]. The generators run with random phases and periodically produce independent dipoles which propagate toward the center of the arena, generating at that location a statistically stationary turbulent flow.

After a statistically steady state is reached, horizontal and vertical velocity fields are measured in the rotating frame thanks to a corotating particle image velocimetry system. Up to 10 000 image pairs are acquired at 1 Hz by a double-frame 2048^2 pixels camera. Homogeneity in the central area of the arena has been carefully characterized by splitting the measured flow in mean and turbulent components using the Reynolds decomposition, $\mathbf{u}(\mathbf{x}, t) = \overline{\mathbf{u}}(\mathbf{x}, t) + \mathbf{u}'(\mathbf{x}, t)$, where

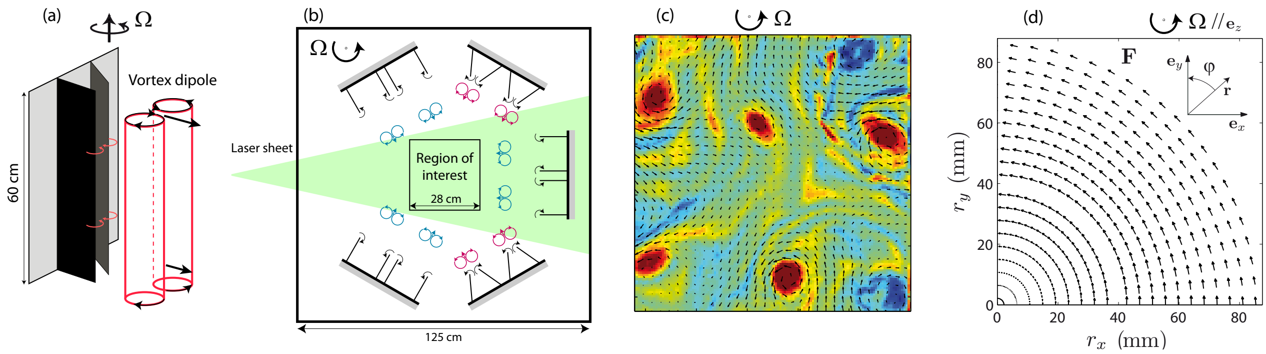

Figure 1. (a) Schematic view of a vortex dipole generator. (b) Top view of the 10 dipole generators placed in the square water tank mounted the platform rotating at $\mathbf{\Omega}$. (c) Typical velocity field measured in the horizontal plane for $\Omega = 12$ rpm. The color maps the corresponding vertical vorticity ω_z. (d) Energy flux density in the horizontal plane for $\Omega = 12$ rpm.

the time average $\overline{\mathbf{u}}(\mathbf{x}, t)$ is computed over the 10 000 velocity fields, from which the total and turbulent kinetic energies, $k_{tot}(\mathbf{x}) = \overline{\mathbf{u}^2}$ and $k_{turb}(\mathbf{x}) = \overline{\mathbf{u}'^2}$, are computed. We find that the spatial fluctuations of $k_{turb}(\mathbf{x})$ are less than 10% over the region of interest of side 28 cm (Fig. 1b). Moreover, the turbulence ratio k_{turb}/k_{tot} is larger than 96% in this area, indicating that our setup successfully produces a nearly homogeneous turbulence in the center of the arena. In this area, the turbulent Reynolds number $Re = u_{rms} L_h / \nu$ is in the range 350–1000, and the Rossby number $Ro = u_{rms}/2\Omega L_h$ in the range 0.04 – 0.4, where u_{rms} is the r.m.s. horizontal velocity and L_h is the horizontal integral scale.

RESULTS

Fig. 1c shows a typical velocity field obtained for a rotation rate $\Omega = 12$ rpm and a frequency of vortex dipole emission of 0.1 Hz for which the excited vortices have an initial rotation rate of 40 rpm. The vorticity field clearly shows a set of strong cyclonic vortices (in red), surrounded by weaker vorticity filaments. The cyclone-anticyclone asymmetry can be quantified by the skewness of vertical vorticity $S_\omega = \langle \omega_z^3 \rangle / \langle \omega_z^2 \rangle^{3/2}$ [9, 10]. For this Reynolds number ($Re \simeq 850$), the asymmetry grows approximately linearly with Ω, and saturates to a maximum of $S_\omega \simeq 2.3$ for $Ro \simeq 0.07$.

The corresponding map of the energy flux density $\mathbf{F}(\mathbf{r})$ in the horizontal plane is shown in Fig. 1d. According to the KHM equation (1), the divergence of this vector field is the source term for the dynamics of the velocity correlation. $\mathbf{F}(\mathbf{r})$ is remarkably invariant with respect to rotation about the z axis, illustrating the excellent degree of axisymmetry of the turbulence generated in our setup. More interesting, $\mathbf{F}(\mathbf{r})$ is almost azimuthal (along $\mathbf{e}_\varphi$), a clear signature of breaking of the mirror symmetry. The orientation of $\mathbf{F}$ is cyclonic (i.e., in the same direction as the background rotation $\mathbf{\Omega}$), and is therefore directly related to the sign of the vorticity skewness. The radial component of $\mathbf{F}(\mathbf{r})$, which primarily contributes here to the energy flux $\Pi(\mathbf{r})$, is surprisingly small compared to this azimuthal component F_φ.

These preliminary results show that our experiment successfully produces a stationary, homogeneous and '*weak-axisymmetric*' turbulence which lacks mirror symmetry with respect to vertical planes as a result of the cyclone-anticyclone asymmetry. This experimental setup provides a unique tool to explore the implication of this additional degree of freedom on the dynamics of the anisotropic energy transfers in rotating turbulence.

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