EXPLICIT ALGEBRAIC MODELS FOR TURBULENT FLOWS WITH BUOYANCY EFFECTS

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Abstract For turbulent flows that are influenced by an active scalar, the Reynolds stresses and scalar flux are coupled in a complicated way, which makes it difficult to model these flows. A framework has been derived for obtaining explicit algebraic Reynolds-stress and scalar-flux models for two-dimensional mean flows with stratification. For the specific case of stably stratified parallel shear flows, the derived model was shown to give good results. As an extension of these results, two more cases are considered: unstable stratification in a horizontal channel and natural convection in a vertical channel.

1. INTRODUCTION

The class of explicit algebraic models, which are derived from the transport equations for the Reynolds stresses $\overline{u_i u_j}$ and scalar flux $\overline{u_i\theta}$, are known to capture more physics than standard eddy-diffusivity models, while being relatively easy to implement. Well-established explicit algebraic models already exist for passive scalars [3, 4]. Deriving such models becomes more complicated for active scalars, e.g. the density when buoyancy forces are presents, such as in atmospheric flows. In these cases, the equations for the Reynolds stresses and scalar flux are mutually coupled, and finding an explicit solution to these equations is not straightforward. Recently, the current authors derived a framework for two-dimensional mean flows from which explicit algebraic models can be obtained for various cases that include buoyancy [2]. Expressed in terms of $a_{ij} := \overline{u_i u_j}/K - \frac{2}{3}\delta_{ij}$ and $\xi_i := \overline{u_i\theta}/\sqrt{KK_\theta}$, where $K := \frac{1}{2}\overline{u_k u_k}$ and $K_\theta := \frac{1}{2}\overline{\theta^2}$, the implicit algebraic equations that need to be solved have the following form:

$$N(a_{kl},\xi_k)\cdot a_{ij} = \mathcal{L}^{(a)}_{ij}(a_{kl},\xi_k), \qquad N_\theta(a_{kl},\xi_k)\cdot \xi_i = \mathcal{L}^{(\xi)}_{i}(a_{kl},\xi_k), \tag{1}$$

in which the left-hand sides contain a scalar nonlinearity in a_{ij} and ξ_i, while the right-hand sides are linear. By temporarily assuming that N and N_θ are known coefficients, the problem becomes linear, and one can search for explicit solutions of the form:

$$a_{ij} = \sum_{k=1}^{M} \beta_k T^{(k)}_{ij}, \qquad \xi_i = \sum_{k=1}^{M'} \lambda_k V^{(k)}_i, \tag{2}$$

where the T^k_{ij} and V^k_i are an appropriate set of tensors and vectors spanning the solution space of (1) and involving the mean strain-rate tensor S_{ij}, the mean rotation-rate tensor Ω_{ij}, the mean temperature gradient Θ_i and the gravitational acceleration g_i. One then obtains a system of 18 linear equations for the coefficients β_k and λ_k, which in principle can be solved for any two-dimensional mean flow. The final step in obtaining the explicit algebraic model is finding expressions for the unknown factors N and N_θ. More details are shown in [2].

2. STABLY STRATIFIED PARALLEL SHEAR FLOWS

A fully explicit algebraic model has been derived from this framework for the specific case of stably stratified parallel shear flows. The detailed expressions for the β_k and λ_k coefficients in this case are given in [2]. The factors N and N_θ turn out to be roots of a sixth-order polynomial, for which no analytical solution exists. However, approximate expressions for these factors have been found, which turn out to give satisfying results specifically for the case of stably stratified parallel shear flow. The explicit algebraic model has been tested for two test cases with stable stratification, namely homogeneous shear flow and channel flow, the results of which are also shown in [2]. Figure 1 compares some results of the channel flow calculations with DNS data obtained by García-Villalba & del Álamo [1] for different values of the friction Richardson number Ri_τ. We can see that there is a good agreement between the model results and the DNS, even for the highest Richardson numbers. One should note that these calculations have been performed within the framework of the low-Re K-ω model proposed by Wilcox [5] and that further corrections are needed to ensure the correct asymptotic behaviour near the wall. Furthermore, the model constants have been calibrated in such a way that no singularities occur in the coefficients β_k and λ_k for these cases.

3. UNSTABLY STRATIFIED PARALLEL SHEAR FLOWS

In order to obtain a model that can be used in atmospheric applications, also cases with unstable stratification need to be considered. We choose to test our model for unstable stratified channel flow. In this case, one can use the same

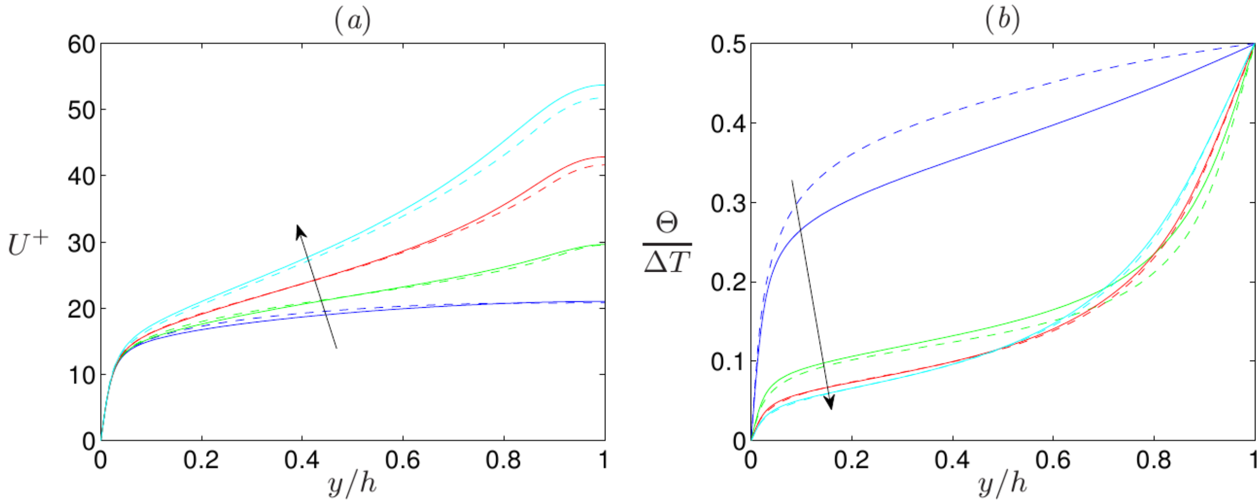

Figure 1. Comparison of the channel flow results (dashed lines) with DNS by García-Villalba & del Álamo [1] (solid lines), for $Re_\tau = 550$ and $Ri_\tau = 0$ (blue), 120 (green), 480 (red), 960 (cyan). The arrows point in the direction of increasing Richardson number. Shown are (*a*) the mean velocity profile and (*b*) the mean temperature profile.

expressions for the β_k and λ_k as was done for stably stratified parallel shear flows, the only difference being the fact that the temperature gradient points downward, i.e. the scalar product $\Theta_k g_k$ changes sign. While the model was found to be nonsingular in the previous case with stable stratification, this change of sign will give a singularity in the case of strong unstable stratification. However, these singularities might only occur at unphysically high Richardson numbers, where no equilibrium solution exists. Furthermore, the previously used expressions for N and N_θ were derived specifically for stably stratified parallel shear flows, and new expressions are needed in the unstably stratified case. Some test runs have been performed with the modified model for unstably stratified channel flow, and the results appear to be qualitatively correct. The next step is to obtain DNS data for this test case so that quantitative comparisons can be done.

4. NATURAL CONVECTION IN A VERTICAL CHANNEL

Another interesting test case is a vertical channel with a temperature difference between the walls, so that natural convection occurs. This case is quite different from the previous two because the temperature gradient is now perpendicular to gravity, i.e. $\Theta_k g_k = 0$, and the important invariant turns out to be $\Theta_k S_{km} g_m$. New expressions for β_k and λ_k have been found from the framework discussed in Section 1, which reveal that singularities might occur, though only on one side of the channel. These singularities need to be investigated further, as well as the required expressions for N and N_θ.

5. CONCLUSIONS AND OUTLOOK

The model derived for stably stratified parallel flows has been shown to give good results compared to DNS data for stably stratified channel flow, and it has a large potential for use in atmospheric applications. Before we can apply the model to atmospheric test cases, the required modifications for unstable stratification need to be investigated in more detail. Furthermore, the interesting case of vertical channel flow can give more insight into the performance of the model in cases with $\Theta_k g_k = 0$, which is a next step towards general two-dimensional mean flows with an arbitrary alignment of the temperature gradient with respect to the direction of gravity.

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