HALL EFFECTS ON ENERGY TRANSFER OF ISOTROPIC MHD TURBULENCE

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Abstract Hall effects on inter-scale energy transfer and energy exchange between the kinetic and magnetic energies in decaying homogeneous and isotropic magneto-hydrodynamic turbulence are studied by means of direct numerical simulations. Energy transfer analysis reveals that the introduction of the Hall term to the induction equation brings about enhancement of the forward energy flux in the momentum equations. The Hall term also changes the spatial structures drastically. Properties of the energy transfer/flux functions which should be responsible to the change of the spatial structures and possibilities to replace the small scales of turbulence by a dissipative sub-grid-scale model for large eddy simulations are discussed.

Introduction

Macroscopic motions of hot plasmas in fusion and solar wind researches are often studied by means of linear stability analysis and nonlinear numerical simulations of single-fluid magneto-hydrodynamic (MHD) equations. However, in association with an increase of the computational resources, the spatial scales resolved in MHD simulations are often finer than the scales allowed in the MHD approximation. For example, the ion skin depth and the ion Larmor radius are the length scales neglected in the MHD approximation. While typical values of them in a heliotron-type fusion experiment device, the large helical device (LHD), are *5*cm and *5*mm, respectively, a full-three-dimensional MHD simulation of the LHD resolves the length of about *1*cm or shorter [1]. Since the small scale dynamics in such simulations are not valid anymore, we need to extended the MHD equations to the scales neglected in the MHD equations and take more detailed physics such as the whistler modes, the kinetic Alfven waves and enhancement of the magnetic reconnections into account. The Hall MHD model is the among simplest extended-MHD equations. While the Hall term provides many new physics to MHD phenomena [2-5], it often restricts time increments in simulations severely due to the whistler modes. We would like to clarify the Hall effects on very small scales and find some appropriate approaches to avoid the excessive restrictions in numerical simulations. Here we study properties of homogeneous and isotropic Hall MHD turbulence by means of direct numerical simulations (DNS) with an intention to highlight small scale spectral properties and possible modeling of the sub-grid-scales for large eddy simulations.

DNS, results and concluding remarks

DNSes of the freely-decaying homogeneous and isotropic turbulence are carried out for the incompressible Hall MHD equations by the use of the pseudo-spectral method and the Runge-Kutta-Gill scheme. The number of the grid points are typically $N^3=1024^3$. The parameter is varied $\varepsilon_H = 0.05, 0.025, 0.0125$ while the resistivity and the viscosity are set $\eta = \nu = 5\times 10^{-4}$. We report that the magnetic energy spectrum is likely to be proportional to $k^{-7/3}$ as have been reported earlier [2,5], while the kinetic energy spectrum is likely to be proportional to $k^{-5/3}$. An interesting observation is that the magnitudes of the kinetic energy spectrum at the small wave numbers decay quickly, while those of the magnetic energy spectrum is kept almost steady, suggesting that the energy exchange between the kinetic and the magnetic energy maintains the magnetic energy spectrum. Aiming to clarify the mechanism of the inter-scale energy transfer and the energy exchange between the kinetic and magnetic spectra in Hall MHD turbulence, we introduce the energy transfer functions and the energy flux functions in the same manner as those for hydrodynamic turbulence: for a system of equations $d\widetilde{U}_k(t)/dt = \widetilde{L}_k(\widetilde{U}_k;t) + \widetilde{N}_k(\widetilde{U}_k;t)$ where $\widetilde{U}_k(t), \widetilde{L}_k(\widetilde{U}_k;t), \widetilde{N}_k(\widetilde{U}_k;t)$ are the shell-averaged energy spectra, the corresponding energy dissipation and transfer spectra, respectively, the flux function is defined as $\widetilde{\Pi}_k(\widetilde{U}_k;t) = \sum_{k'>k} \widetilde{N}_{k'}(\widetilde{U}_k;t)$. In Fig. 1(a) and (b), the energy flux functions associated with the advection terms of the momentum equations and the induction equations at a late stage of the time evolutions are shown, respectively. It is clearly observed in Fig. 1(a) that the energy flux from the large to the small scales in the kinetic energy spectrum is enhanced as the Hall parameter increases, while the advective energy flux in the magnetic energy equation in Fig. 1(b) is not necessarily enhanced but the peak position of the flux is shifted to the smaller scales by the increased Hall parameter. It appears that the introduction of the Hall term changes not only the inter-scale energy transfer in the magnetic energy spectrum but also the energy transfer in the kinetic energy spectrum, although the analysis for one-time-snapshot is insufficient and we need further analysis including the time-average over the statistically steady state. The enhancement of the energy transfer in the momentum equation suggests that the small-scale properties in velocity field are strongly modified from those in the single-fluid MHD turbulence. Further analysis including the flux functions associated with the Lorentz force term in the momentum equation and the magnetic stretching term in the induction equations, which are closely related with the energy exchange between the kinetic and magnetic energy, or the dynamo action, will also be shown in the presentation.

In Figure 2, the isosurfaces of the enstrophy density (square of the rotation of the velocity vector) and the current density are shown. While the isosurfaces of the enstrophy density field are often sheet-like and move with the current sheets in the single-fluid MHD turbulence [2], the isosurfaces in Fig. 2 is clearly tubular and resemble to those in hydro-dynamic turbulence [6]. We also observe that the introduction of the Hall term causes formation of more fiber-like current structures than those in the single-fluid MHD turbulence. In the presentation, we would like to show more detailed views of the spatial structures in the relation with the energy fluxes which are modified by the introduction of the Hall term. We also try to evaluate local/non-local natures of the nonlinear couplings and applicability of the traditional sub-grid-scale models to Hall MHD turbulence, for the sake of carrying out of large eddy simulations of Hall MHD turbulence.

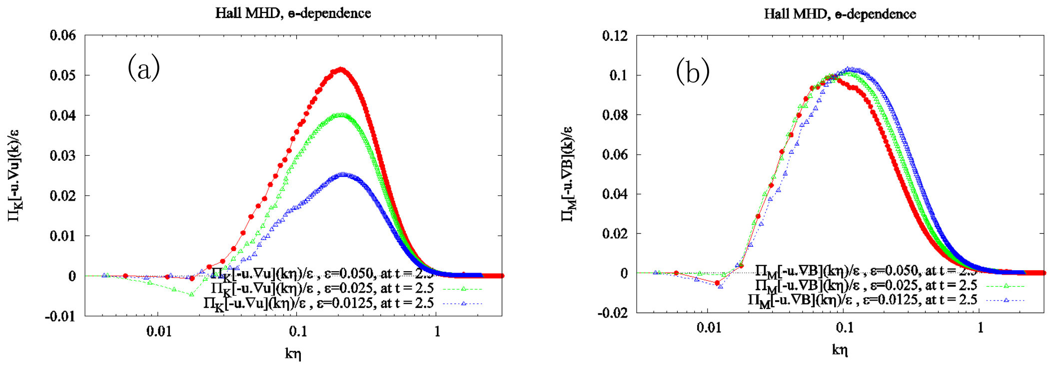

Figure 1. Energy flux functions of the advection terms in (a) the momentum equations and (b) the induction equations. The abscissa is normalized by the Kolmogorov length scale η_K and the vertical axis is normalized by the energy dissipation rate.

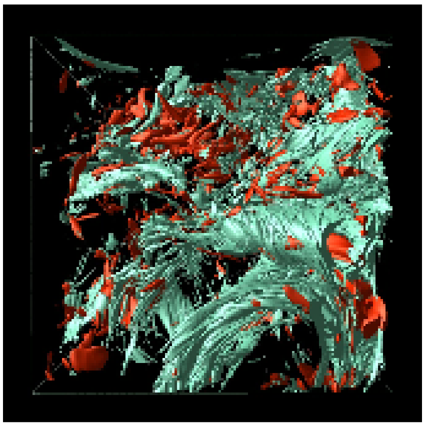

Figure 2. Isosurfaces of the enstrophy density (red) and the current density (cyan) in the DNS of Hall MHD turbulence with the Hall parameter $\varepsilon_H = 0.05$. The thresholds for the isosurfaces are roughly given by the two times root-mean-square above the mean values of the enstrophy and the current density, respectively.

References

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