BIFURCATIONS FROM DOUBLE-LAYERED STREAMWISE-INDEPENDENT VORTEX FLOW IN ROTATING PLANE COUETTE FLOW

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Abstract Two new classes of three-dimensional wavy vortex flows are found in rotating plane Couette flow. One of the classes bifurcates subharmonically from streamwise-independent vortex flow which sets in when the Taylor number exceeds its *second* critical number. Mirror-symmetry about the planes normal to the parallel boundaries characterizes the flow. The other class also bifurcates from the same streamwise-independent vortex flow and possesses the mirror-symmetry as well but shift-reflection and shift-rotation symmetries of the first class are absent. The both new classes inherit a double-layered structure of the streamwise-independent vortex flow, and exist off the bifurcation route known so far.

INTRODUCTION

During the past few decades, significant insights have been gained in the transition to turbulence by applying the theory of nonlinear dynamical systems to flows which exhibit sequences of bifurcations [1]. In particular, a rich tapestry of transition sequences have been observed by studying the competition between the induced body forces, such as a system rotation, and shear-induced instabilities both theoretically [2] and experimentally [3, 4]. Here, we report on new results in rotating plane Couette flow whereby the existence of two separate bifurcation routes is demonstrated.

MODEL

We consider an incompressible viscous fluid motion between two parallel planes of infinite extent which moves in the opposite directions with the speed U_0, subject to a spanwise rotation Ω_0. Disturbed velocity $\boldsymbol{u}$ and pressure p from the basic state are governed, in the non-dimensional form, by the equations of continuity and momentum conservation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \partial_t \boldsymbol{u} + + \boldsymbol{U}_B \cdot \nabla \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{U}_B + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \Omega \boldsymbol{j} \times \boldsymbol{u} = -\nabla p + \frac{1}{Re} \nabla^2 \boldsymbol{u}, \tag{1}$$

with the no-slip condition at the boundaries, $z = \pm 1$, where $\boldsymbol{U}_B$ is the basic velocity given by $\boldsymbol{U}_B(z) = -z\boldsymbol{i}$. The unit vectors in the streamwise and the spanwise directions, corresponding to the x-, and y-directions, respectively, are denoted by $\boldsymbol{i}$ and $\boldsymbol{j}$. We are interested in the development of disturbances which is controlled by the Reynolds number, $Re = \frac{U_0 d}{\nu}$, and the rotation number, $\Omega = \frac{2\Omega_0 d}{U_0}$, where d is the half gap and ν is the kinematic viscosity.

NUMERICAL METHODS

A typical component, q, of the disturbances is expressed as follows:

$$q(x, y, z, t) = \sum_{l=-L}^{L} \sum_{m=-M}^{M} \sum_{n=-N}^{N} q_{lmn}(t) \exp[im\alpha x + in\beta y] T_l(z), \tag{2}$$

where α and β are the wavenumbers in the streamwise and the spanwise directions, respectively, and $T_l(z)$'s are the modified Chebyshev polynomials to satisfy the no-slip boundary condition. After the discretization of the equations is done by the collocation method, the resulting algebraic equations for the amplitudes, q_{lmn}'s, are solved by Newton method.

RESULTS

We first review the well-known bifurcation route which originates when the basic state loses stability at the critical Taylor number $Ta = \Omega(Re - \Omega) = 106.735$. Streamwise-independent flow (TV_1), corresponding to the Taylor vortex flow in the Taylor-Couette system, follows, and wavy vortex flow (WVF) bifurcates from TV_1. The class of this WVF is classified as $\mathscr{A}_2$ in [5]. WVF with some wavenumber pair (α, β) exists at $\Omega = 0$ (plane Couette flow) [6] as shown in Figure1 (a). Not shown in the figure is the solution branch, called *t*wists, which belongs to $\mathscr{A}_1$ and appears at larger Ω and Re [5].

Our present analysis reveals that there exists another bifurcation route from the basic state. Namely, streamwise-independent vortex flow (TV_2) with a double-layered vortex structure bifurcates from the basic state at the *second* critical Taylor number $Ta = 1100.650$ and two new classes of three-dimensional flows bifurcate from TV_2 as shown in Figure 1(b). Although these classes were classified as $\mathscr{A}_0$ (or *Ribbon*) and $\mathscr{A}_3$ in [5], they have not been detected so far. The bifurcation of $\mathscr{A}_0$

(a) (b)

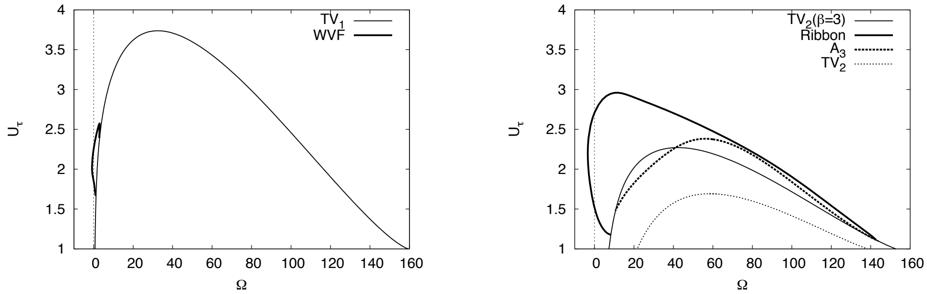

Figure 1. The conventional bifurcation route (a) and the new bifurcation route (b). $R = 160$. $(\alpha, \beta) = (0.9, 1.5)$ except for TV_1 in (a) and TV_2 (thin dotted curve) in (b) for which $(\alpha, \beta) = (0.0, 1.5)$, another TV_2 (thin solid curve) in (b) for which $(\alpha, \beta) = (0.0, 3.0)$ and $\mathscr{A}_3$ in (b) for which $(\alpha, \beta) = (1.8, 3.0)$.

is subharmonic. The flow structure of the class $\mathscr{A}_0$ is characterized by a mirror-symmetry about the plane $y = \pi/\beta$ (see Figure 2(a)). The class $\mathscr{A}_3$ keeps the mirror-symmetry as well (see Figure2(b)), but shift-reflection and shift-rotation symmetries of $\mathscr{A}_0$ are not present.

(a) (b)

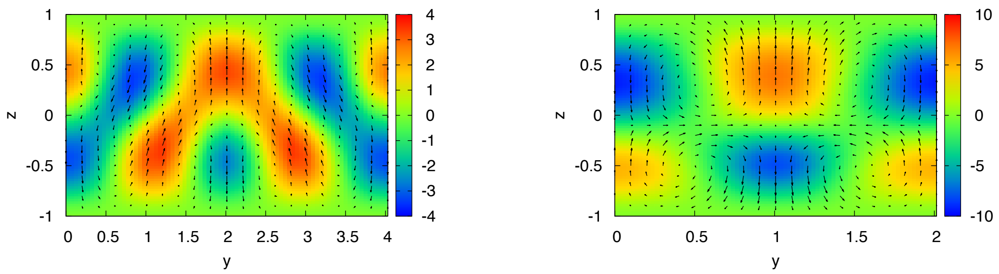

Figure 2. The cross-sectional velocity structure at $x = (1/4)(2\pi/\alpha)$. $R = 100$. The colour code indicates the streamwise veolcity component. (a): $\mathscr{A}_0$ (or *Ribbon*) with $(\alpha, \beta) = (0.8, 1.56)$ at $\Omega = 13$. (b): $\mathscr{A}_3$ with $(\alpha, \beta) = (1.6, 3.12)$ at $\Omega = 20$.

SUMMARY

New solution classes, $\mathscr{A}_0$ and $\mathscr{A}_3$, are found in rotating plane Couette flow on a bifurcation route different from the one known so far. Both classes inherit the double-layered vortex structure of TV_2 they have bifurcated from. As in the case of WVF, the class $\mathscr{A}_0$ reaches $\Omega = 0$ for some (α, β) as demonstrated in Figure 1(b). We shall show the connection between the solutions of the class $\mathscr{A}_0$ at $\Omega = 0$ and the so-called hairpin vortices [7, 8] in plane Couette flow.

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