

OPTIMAL AMPLIFICATION OF STREAMWISE STREAKS IN PLANE JETS AND THEIR STABILIZING EFFECT ON THE INFLECTIONAL INSTABILITY

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Abstract Optimal transient energy growths supported by the plane Bickley jet are computed for a set of spanwise wavenumbers and Reynolds numbers. It is shown that the maximum energy amplification is proportional to the square of the Reynolds number. The computed optimal streamwise vortices are then used to efficiently force finite amplitude streaks that are shown to stabilize the jet's powerful inflectional instability, which is clearly relevant for a number of applications in the control of free shear flows.

In high Reynolds number shear flows, low energy streamwise vortices can be converted into high energy streamwise streaks via the lift-up effect. The energy growth involved in this process is transient and is related to the non-normality of the linearized Navier-Stokes operator ruling the evolution of small perturbations to the basic flow. Most of the interest in streaks transient growth has been attracted by its role in laminar-turbulent transition in linearly stable flows and by its role in self-sustained processes in wall-bounded turbulent flows. Recent research has, however, also demonstrated that streamwise streaks can be artificially forced to efficiently manipulate flows. In particular, it has been shown that the primary viscous instability in the Blasius boundary layer can be stabilized by forcing suitable streamwise streaks [1] and transition to turbulence therefore delayed [3]. This methodology has then been extended to the manipulation of mean flows for turbulent drag reduction [2, 5, 6].

The scope of the present study is to ascertain if the control-by-streaks approach can be extended to the stabilization of inflectional instabilities. To this end we consider as basic flow the prototypical Bickley jet profile $U_B(y) = U_0 \operatorname{sech}^2(y/\delta_B)$ that is well known to be inviscidly unstable. In order to force the desired streaks using the minimum input energy, the optimal energy growth $G(t) = \max_{\mathbf{u}_0 \neq 0} \|\mathbf{u}(t)\|^2 / \|\mathbf{u}_0\|^2$ is computed for streamwise uniform perturbations with same symmetry as the basic flow about the $y = 0$ plane and for a set of selected spanwise wavenumbers β and Reynolds numbers $Re = 2\delta_B U_0 / \nu$.

Streamwise uniform perturbations are linearly stable, but can undergo transient energy amplifications, as seen in the left panel of fig. 1. We find that for $Re \gtrsim 50$ the maximum amplification $G_{max}(\alpha = 0, \beta, Re) = \max_t G(t, \alpha = 0, \beta, Re)$ satisfies the scaling $G_{max}(\alpha = 0, \beta, Re) = Re^2 \hat{G}_{max}(\beta)$ as seen in the right panel of fig. 1. This scaling, identical to the one found for wall-bounded shear flows [4], is very promising for flow control applications, indicating that the control amplification is more efficient at large Re and that the optimal perturbations, such as the one shown in the left panel of fig. 2, do not change with Re .

To assess the influence of the streamwise streaks on the inflectional instability we follow the same approach used in previous investigations of wall-bounded shear flows [1, 6]. A family of streaky basic flows is generated by using linearly optimal streamwise vortices with finite amplitude A_v as initial condition in fully nonlinear Navier-Stokes simulations. The developing nonlinear streaks profiles $U(y, z)$ are then extracted at the time t_{max} at which the maximum amplitude is reached in the linear case. The streak amplitude is defined as $A_s = \{\max_{y,z}[U(y, z) - U_B(y)] - \min_{y,z}[U(y, z) - U_B(y)]\} / (2U_0)$. The 2D plane jet, taken as baseline at $Re = 50$ and labeled case **A**, corresponds to $A_s = 0$, while streaky basic flows **B** and **C** respectively correspond to $A_s = 0.6$ and $A_s = 0.75$.

The linear impulse response evolving on top of the considered (frozen) basic flows **A**, **B** and **C** is then computed by direct numerical simulation in order to test their linear stability. The temporal evolution of the mean kinetic energy density E' of the impulse response is reported in the right panel of fig. 2. Since all the modes are excited by the nearly-impulsive initial condition, any unstable mode would emerge after an initial transient. The most unstable mode of the 2D Bickley jet (case **A**) is seen to emerge quickly, for $t \gtrsim 10$, beyond which its energy is exponentially amplified with growth rate $\sigma \simeq 0.169$. In the presence of nonlinear streaks the growth rate is strongly reduced: $\sigma \simeq 0.095$ for case **B** and $\sigma \simeq 0.035$ for case **C**.

The results reported above show that large amplitude streamwise streaks can be efficiently forced in jets by optimal streamwise vortices. Large amplitude streamwise streaks have a stabilizing effect on the jet's inflectional instability. Forcing the stabilizing streaks with optimal perturbations ensures that the energy of the forced streamwise vortices is very small, unlike the energy of vortices forced by tabs or vortex-generators considered in previous investigations. Also, because of the translational invariance of the growth rates, these results are immediately extended to the case of wakes with velocity profile $U(y) = U_\infty - U_B(y)$.

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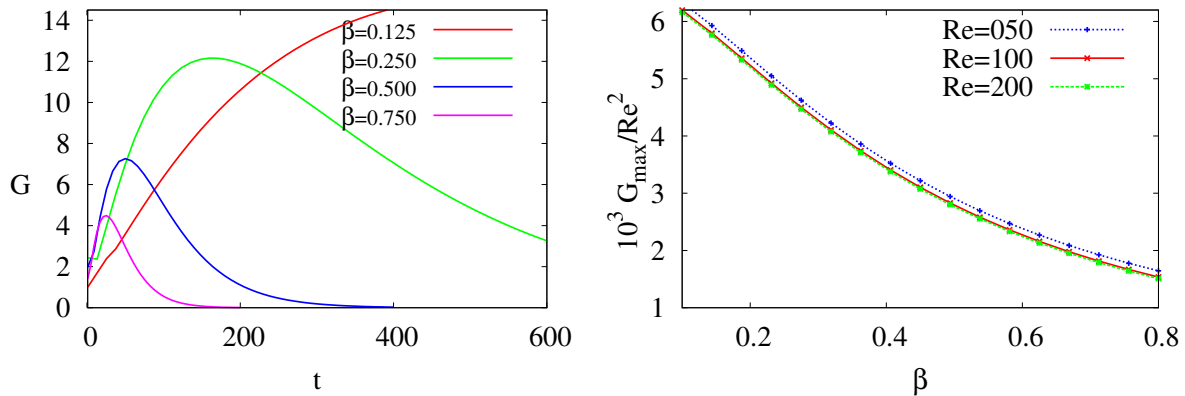


Figure 1. Left panel: Optimal energy growth curves $G(t)$ of streamwise uniform ($\alpha = 0$) perturbations for selected spanwise wavenumbers β at $\text{Re} = 50$. Right panel: rescaled maximum growths versus β for three selected Re .

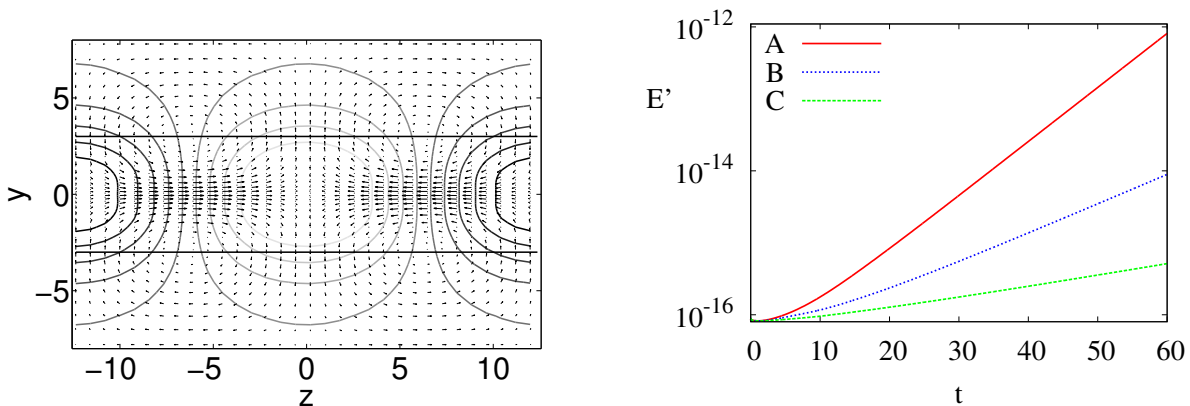


Figure 2. Left panel: Cross-stream view of the optimal initial streamwise vortices (arrows) and optimally amplified streamwise streaks (contour lines) for $\beta = 0.25$. The two horizontal lines correspond to the shear layer boundary where $U(y)=0.01$. The optimal initial streamwise vortices are approximately centered on this boundary and induce alternated low and high speed streaks. Right panel: Temporal evolution of the impulse response energy E' evolving on top of the 2D Bickley jet A and on the increasingly streaky jets B and C. The presence of streaks stabilizes the unstable inflectional mode.

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