TRANSITION TO TURBULENCE IN STRATIFIED SHEAR FLOW THROUGH AN INCLINED SQUARE DUCT

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Abstract We describe experiments on buoyancy-driven shear flow in an inclined duct. The duct connects two reservoirs containing fluids of different densities and the exchange flow along the duct produces a shear flow whose properties depend on the density difference and the angle of inclination. We explore the various flow regimes and determine the critical condition for the transition from the turbulent to intermittent regime.

INTRODUCTION

Turbulence in a pipe is the canonical example of turbulent shear flow. The primary research focus has been unstratified flows in a circular pipe and under these conditions, the transition to turbulence and the properties of the turbulence are governed by a single control parameter, the Reynolds number Re. As Re increases the flow transitions from laminar through a spatio-temporal intermittent laminar/turbulent regime until the turbulence becomes fully-developed [2].

Here we consider the transition to turbulence in a stratified square duct. The duct joins two reservoirs containing fluids of different densities $\Delta\rho$ and can be inclined at an angle θ to the horizontal. In addition to the Reynolds number $Re \equiv UH/\nu$, where H is the height and width of the duct, the flow is also governed by a second control parameter, the Richardson number $Ri \equiv g\cos\theta\Delta\rho L/\rho U^2$, where L is the length of the interface and U is the average velocity in each layer [3]. This second control parameter for the dynamics of turbulent stratified shear flow to be examined in $Re - Ri$ space [1].

EXPERIMENTS

In this experiment a water tank is partitioned into two sections, both containing salt water with different densities. A square duct, which may be inclined at an angle θ to the horizontal, passes throughout the central divide of the tank and connects the two sections of the tank, which act as large reservoirs. The flow is initiated by opening one end of the duct and maintained until the fluid entering each reservoir begins to affect the input at either end of the duct. The flow is characterised as a function of the inclination θ and the density difference $\Delta\rho$ between the two sections of the tank. In these experiments we explored the flows in the range $0^\circ \leq \theta \leq 3^\circ$ ($\theta > 0$ means that the duct is inclined up towards the dense reservoir – see figure 1) and $10 \leq \Delta\rho \leq 210$ [kg m^{-3}]. Flow visualisation was made using shadowgraph and fluxes were determined by measurements of the reservoir densities.

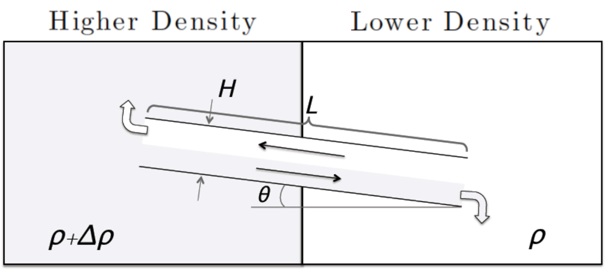

Figure 1. Experimental set-up.

RESULTS

The buoyancy forces establish a flow with the light fluid flowing uphill in the upper part of the duct and the dense fluid flowing downhill in the lower part. Between these counter flowing layers there is an interfacial region (IR) which takes different forms depending on the parameters (figure 2(a)). At high angles or large density differences, the IR is strongly turbulent with three-dimensional structures of Kelvin-Helmholtz type across its whole width. Mass is transferred from the lower layer directly to the upper layer through eddies that span the entire thickness of the IR. As the density difference and/or angle decrease, the turbulence becomes less intense. At a critical density difference and angle combination, a spatio-temporal intermittent regime develops where turbulent bursts and relaminarization events occur. Both KH and Holmboe type modes appear in this regime and the IR has a complicated structure consisting of thinner layers of high density gradients within it. These layers and the instability modes display significant variability in space and time. At even smaller angles and density differences the flow is essentially laminar, with a relatively sharp interface supporting Holmboe-type wave modes with the occasional breaking event. Shadowgraph images of these flow regimes are shown in figure 2.

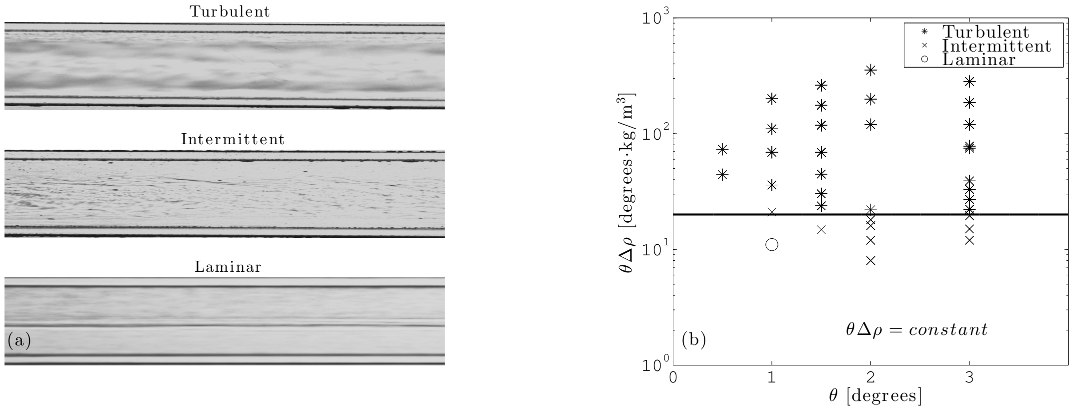

Figure 2. (a) Shadowgraph images of the three primary flow regimes. (b) Phase space for flow dependence on duct incline angle θ and the density difference $\Delta\rho$.

Figure 2(b) shows the location of the flow regimes in the phase plane $\theta\Delta\rho$ vs. θ. As the density difference is decreased the flow transitions from turbulent to intermittent behaviour occur at a constant value of $\theta\Delta\rho \sim 20$ [degrees $\cdot$ kg m^{-3}]. The mass flux along the duct was measured for the cases $\theta = 0, 1.5°$ and the results are shown in figure 3(a). The slope on the log-log plot is very close to 1.5, consistent with a velocity that scales on $\sqrt{\Delta\rho}$ as expected from buoyancy-driven flow. Measurements of the thickness h of the turbulent interface show that $h \sim \theta^{1/3}$ (figure 3(b)).

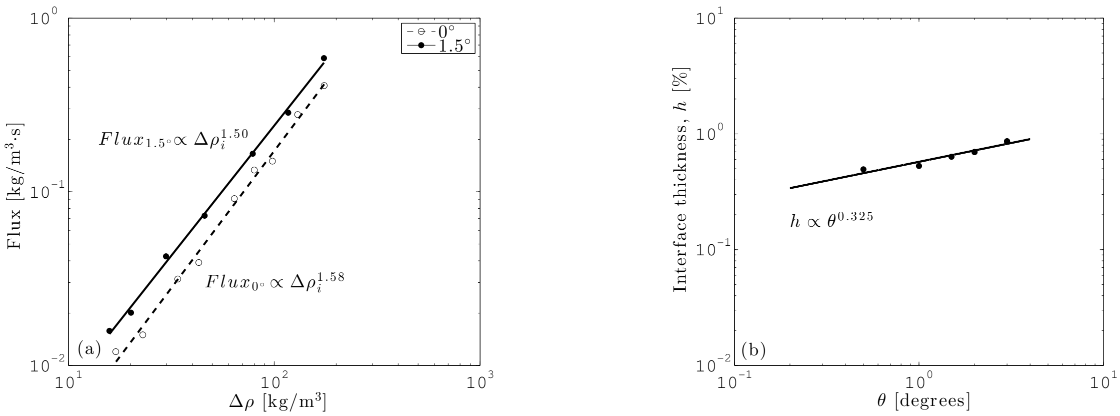

Figure 3. (a) Mass flux through duct for $\theta = 0, 1.5°$. (b) Interface thickness h/H in the turbulent region (large $\Delta\rho$) as a function of θ.

DISCUSSION

We have examined the flow regimes associated with buoyancy-driven flow in an inclined duct and observed three flow regimes. The transition from the fully turbulent regime to an spatio-temporal intermittent turbulent/laminar regime occurs at a critical value of $\theta\Delta\rho$. This is consistent with the critical buoyancy Richardson number $R_B = Re/Ri_g$ of Brethouwer et al. 2007 [1] provided $Ri_g \equiv g\Delta\rho h/\rho U^2$ is interpreted as the gradient Richardson number across the IR based on its thickness h. Then, since $h \sim \theta^{1/3}$, constant Ri_g implies $U \sim \sqrt{\Delta\rho}\theta^{1/6}$ and $R_B \sim U^3/\Delta\rho \sim \sqrt{\theta\Delta\rho}$.

References

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