ON THE VALIDITY OF THE LAW OF THE WALL

Gary N. Coleman[1] & Philippe R. Spalart[2]
[1]*Computational AeroSciences Branch, NASA Langley Research Center, MS 128, Hampton, VA, 23681 USA*
[2]*Boeing Commercial Airplanes, PO Box 3707, Seattle,WA 98124 USA*

Abstract Existing and new direct numerical simulation (DNS) results are used to examine the effect of flow Reynolds number and pressure gradients on the law of the wall for incompressible wall-bounded turbulence. We find no evidence from DNS against a universal law of the wall: as the Reynolds number is increased, all sources agree precisely over an increasing range of z^+ on the function U^+ versus z^+. This includes boundary layers, pipes and channels, and Couette-Poiseuille flows with mild but non-negligible, favorable and adverse, pressure gradients. We consider the law of the wall to be firmly established up to $z^+ \approx 400$. On the other hand, a logarithmic region (in the sense of constant logarithmic slope equal to the inverse of the von Karman coefficient κ) is not present for the Reynolds numbers accessible to DNS today. A constant slope roughly over the range [150,400] is recovered by introducing a shifted origin for z^+, but the required value of κ is then near 0.35, which is outside the range of recent proposals from experiments, namely [0.38,0.42], let alone the range that prevailed for decades, namely [0.40,0.41].

The objective of this study is to investigate the effect of geometry, Reynolds number and streamwise pressure gradient on the law of the wall for the mean velocity. We distinguish between the law of the wall and the logarithmic law, with the former stating only that in the near-wall region U^+ depends only on z^+. This is done by comparing results from previous DNS of the Ekman layer [6], plane channel [1], zero-pressure-gradient (ZPG) boundary layer [5] and Couette-Poiseuille flow [2], as well as measurements from the Superpipe experiment [4]. We also include a new higher-Reynolds number DNS of the Ekman layer, for which $R_G = GD/\nu = 2828$ (where G is the geostrophic wind speed, $D = (2\nu/f)^{1/2}$ the viscous depth, f the Coriolis parameter, and ν the kinematic viscosity). This simulation is similar to those described in [6] in terms of numerical approach and parameters, and flow geometry. It required $N_x \times N_y \times N_z = 1344 \times 4032 \times 273 \approx 1.5 \times 10^9$ spectral collocation/quadrature points.

The good agreement, up to $z^+ \approx 300$, of the results from the favorable-pressure-gradient (FPG) cases (channel, pipe and Ekman) shown in figure 1a, implies that in this region the law of the wall is unaffected by lateral surface curvature, system rotation, the opposite wall, or the magnitude of the pressure gradient (inversely proportional to Reynolds number). The total magnitude Q of the mean horizontal velocity vector is shown here, to account for the mean three-dimensionality of the Ekman profile. A more discriminating measure of the agreement is given in figure 1b, where the 'Karman measure' $\kappa_M = \mathrm{d}(\ln z^+)/\mathrm{d}Q^+$ is plotted. Recall that κ_M will be constant and equal to the von Karman constant if/when the mean velocity exactly satisfies the logarithmic law. Also included in figure 1b is the ZPG DNS data from Schlatter & Örlü [5], which agrees well with the FPG cases for $z^+ < 200$ – pointing to the insensitivity to pressure gradient of the $\mathrm{d}U^+/\mathrm{d}z^+$ versus z^+ relationship.

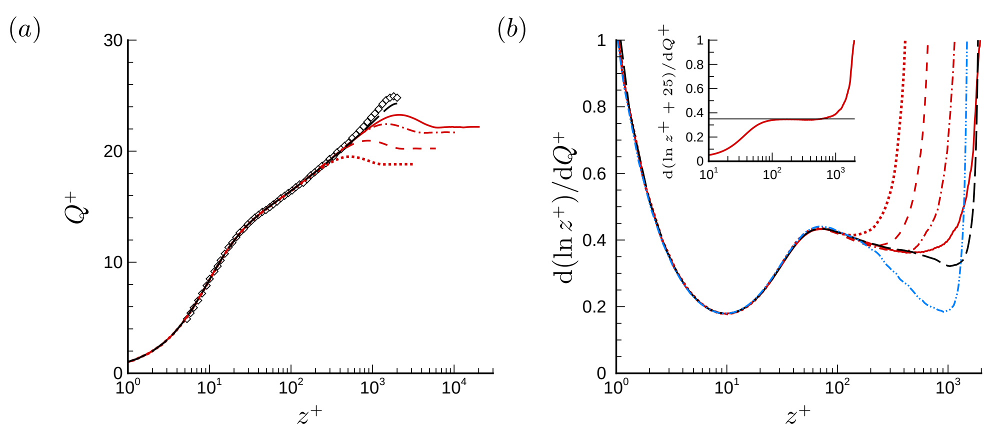

Figure 1. Mean velocity profiles from DNS of turbulent Ekman layer: · · · · · ·, $R_G = 1000$ [6]; - - - - -, $R_G = 1414$ [6]; — · — · —, $R_G = 2000$ [6]; ———, $R_G = 2828$ (new); — — —, $R_\tau = 2003$ plane-channel DNS [1]; – ·· – ·· –, $R_\theta = 4300$ zero-pressure-gradient boundary-layer DNS [5]; ◇ (in a only), $R_D = 74\,345$ Superpipe experiment [4]. Velocity magnitude $Q = (U^2 + V^2)^{1/2}$, where U and V are components in planes parallel to the wall. Horizontal line in inset in b corresponds to $\kappa = 0.35$.

The downward drift of κ_M, roughly from 0.42 at $z^+ = 100$ to 0.36 at $z^+ = 500$ seen in figure 1b indicates that the traditional prediction that the mean velocity satisfies the logarithm relation $\mathrm{d}Q^+/\mathrm{d}\ln z^+ = 1/\kappa$ in this range is not valid to the desired level: we certainly hope to determine the second digit of the Karman constant, and as the evidence stands we can only guess the level of the asymptote, which apparently cannot be reached before $z^+ \approx 1000$ at the least. This has implications for wall-functions used with turbulence models for CFD, but more importantly for the soundness of turbulence theory.

Intriguing behavior is observed when a virtual origin of $\Delta z^+ = 25$ is used in the logarithm relation, which becomes $U^+ = \log(z^+ + \Delta z^+)/\kappa + C$; see the figure 1$b$ inset. This shift keeps the level between 0.34 and 0.35 over the range $[100, 700]$, which is certainly wide. Unfortunately, the value $\kappa \approx 0.35$ is much smaller than most observers would be willing to accept, in light of measurements. We consider such a shift or "virtual origin" to be physically plausible, but its acceptance will depend on establishing (1) that it amounts to more than "making the inflection point horizontal" (until far higher Reynolds numbers are reached), (2) that it does not change the nature of the law-of-the-wall argument, and (3) some degree of compatibility with experimental findings. Community consensus on all these points would be difficult to obtain.

The influence of the sign of the pressure gradient (or more fundamentally, of the slope of the shear-stress profile) upon the law of the wall can be studied better in Couette-Poiseuille flow, since one wall corresponds to an FPG, the other an APG (figure 2a). The star (FPG) and cross (APG) symbols shown in figure 2 were taken from the DNS of Johnstone et al.[2]. Both the FPG and APG profiles are consistent with the earlier results, in that their κ_M agree well with the other cases below $z^+ \approx 100$ (figure 2b), with this limit set by the relatively low Reynolds number and effective PG ratio of that simulation. Preliminary results for a case whose two sides are closer to ZPG than in [2] are also shown (solid and open circles for FPG and APG, respectively). The Karman measure again supports the notion that the law of the wall does not depend upon pressure gradient, at least to levels of the order of 0.0005 in wall units. This would make it more general than predicted by the theory. Here also, consensus is not present, and the simulations that could establish it will be vastly more expensive than those possible today.

At the conference, new Couette-Poiseuille DNS will be presented with Reynolds numbers up to roughly double those considered in [2], for various combinations of pressure gradient and wall velocity U_w, chosen such that R_τ and p^+ are independently varied. We are also hoping for new DNS channel-flow results from colleagues, and for a better understanding of recent experimental pipe-flow results that indicate lower κ values [3] than from the earlier measurements [4]; both would benefit the debate to which this paper aims to contribute.

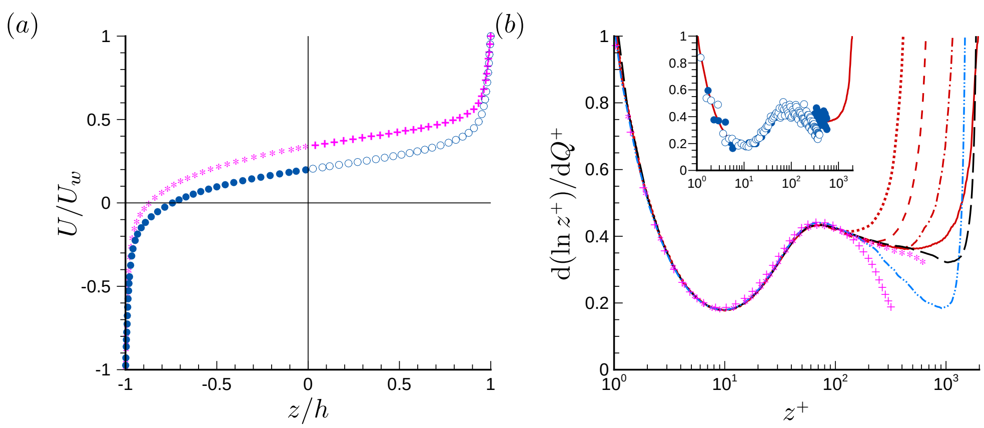

Figure 2. Mean velocity from DNS of Couette-Poiseuille flow: $*$, FPG side ($p^+ = -0.00057$, $R_\tau = 627$) [2]; $+$, APG side ($p^+ = +0.0037$, $R_\tau = 336$) [2]; $\bullet$, FPG side ($p^+ = -0.00046$, $R_\tau = 575$) (new); $\circ$, APG side ($p^+ = +0.00133$, $R_\tau = 400$) (new). Other symbols in b as in figure 1.

We are indebted to Dr. Roderick Johnstone for performing the Ekman-layer and Couette-Poiseuille DNS.

References

[1] S. Hoyas and J. Jiménez. Scaling of the velocity fluctuations in turbulent channels up to Re$_\tau$=2003. *Physics of Fluids* **18**: 011702, 2006.

[2] R. Johnstone, G. N. Coleman and P. R. Spalart. The resilience of the logarithmic law to pressure gradients: evidence from direct numerical simulation. *Journal of Fluid Mechanics* **643**: 163–175, 2010.

[3] I. Marusic, J. P. Monty, M. Hultmark and A. J. Smits. On the logarithmic region in wall turbulence. *Journal of Fluid Mechanics* **716**: R3, 2013.

[4] B. J. McKeon, J. Li, W. Jiang, J. F. Morrison and A. J. Smits. Further observations on the mean velocity in fully-developed pipe flow. *Journal of Fluid Mechanics* **501**: 135–147, 2004.

[5] P. Schlatter and R Örlü. Assessment of direct numerical simulation data of turbulent boundary layers. *Journal of Fluid Mechanics* **659**: 116–126, 2010.

[6] P. R. Spalart, G. N. Coleman and R. Johnstone. Retraction: Direct numerical simulation of the Ekman layer: A step in Reynolds number, and cautious support for a log law with a shifted origin. *Physics of Fluids* **21**: 109901, 2009.