ENERGY CASCADE AND SCALING IN SUPERSONIC TURBULENCE

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Abstract An exact relation for structure functions in isothermal compressible turbulence [6] is verified using data from a large-scale three-dimensional turbulence simulation at Mach 6 [13].

INTRODUCTION

Supersonic turbulence is believed to play a key role in extreme astrophysical and terrestrial environments, e.g., regulating star formation in molecular clouds [8], feeding supermassive black holes [9], creating clumpy structure in hot winds from Wolf-Rayet stars [16], providing the key to reading records of ancient asteroid impacts [7], controlling air entrainment in high-pressure volcanic eruptions [17], and affecting fuel mixing and combustion efficiency in scramjet engines [10]. Compared to incompressible turbulence, highly compressible turbulent flows are more complex due to nonlinear coupling of the velocity, density, and pressure fields. Shock waves and vortex sheets change the topology of interimittent dissipative structures in supersonic turbulence [18]. A universal scaling of the mass-weighted velocity $\rho^{1/3}\boldsymbol{u}$ was demonsrated in numerical experiments [13, 14] and independently confirmed in [12, 19, 20], indicating, by dimensional arguments, the presence of an inertial cascade. More recently, analytical scaling relations for compressible turbulence were derived and analyzed [5, 6, 21, 4] and the existence of intermediate scaling range dominated by inertial dynamics was demonstrated rigorously based on coarse-graining [1, 3, 2]. This contribution reports on the verification of one of these new relations [6] with data from a Mach 6 simulation [13] and on phenomenology that follows from these results.

SCALING RELATION

An isotropic version of the new relation [6, 4] can be written in a symbolic form as

$$S(r) + F_{\parallel}(r) = -\frac{4}{3}\varepsilon r, \tag{1}$$

where $S(r)$ represents compressible source terms, including pressure dilatation, and $F_{\parallel}(r) = \langle[\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u} + 2\delta\rho\delta e]\,\delta u_{\parallel} + \tilde{\delta}e\delta(\rho u_{\parallel})\rangle$ is the longitudinal component of the flux of total energy density $E \equiv \rho u^2/2 + \rho e$, with ρ being the fluid density, $\boldsymbol{u}$ the velocity, $u_{\parallel} \equiv \boldsymbol{u}\cdot\boldsymbol{r}/r$ its longitudinal component, $e \equiv c_{\rm s}^2 \ln(\rho/\rho_0)$ the compressional energy, $c_{\rm s}$ the sound speed, ρ_0 the mean density, δ indicating differences corresponding to the lag $\boldsymbol{r}$, $\tilde{\delta}e \equiv e(\boldsymbol{x}+\boldsymbol{r}) + e(\boldsymbol{x})$, $\langle\ldots\rangle$ denoting an ensemble average, and $\varepsilon \approx \langle\rho\boldsymbol{u}\cdot\boldsymbol{a}\rangle$ being the mean kinetic energy density injection rate by a large-scale external acceleration $\boldsymbol{a}$. As a primitive form of Kolmogorov's four-fifth law [11], $\rho_0\langle(\delta\boldsymbol{u})^2\delta u_{\parallel}\rangle = -4\varepsilon r/3$, relation (1) follows from the constraint imposed on solutions to the isothermal Navier-Stokes system by the presence of an ideal invariant associated with the total energy conservation $\int_V E d^3x = const.$

EXPERIMENTAL VERIFICATION

To evaluate relation (1), we used data from a simulation of homogeneous isotropic turbulence at an r.m.s. Mach number of 6 driven with an external large-scale acceleration [13]. In this study, we analyzed 60 full data snapshots at a grid resolution of 1024^3 evenly distributed over three flow crossing times representing a statistically stationary state of the system. Fig. 1 shows that relation (1) is satisfied quite well (thick solid black and thin solid grey lines follow each other nicely) in the range of scales $r/L \in [0.03, 0.1]$, where the inertial interval is expected in numerical experiments at this resolution (L is the periodic computational domain size). The contributions from the flux (black dashed line) and source (grey solid line) terms have opposite signs and both show roughly linear scaling with r. This is consistent with results of [3, 2] indicating that the sources act primarily on the largest scales, where the external force is active. The inertial term, $\langle\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u}\,\delta\boldsymbol{u}_{\parallel}\rangle$, is responsible for the dominant contribution to the energy flux at Mach 6. More detail can be found in [15].

CONCLUSIONS

We demonstrated that the analytical relation (1) for compressible isothermal turbulence provides a good approximation to numerical results. Our analysis supports a Kolmogorov-like picture of the inertial energy cascade in supersonic turbulence, with the dominant contribution from nonlinear advection, previously discussed on a phenomenological level in [13] and more recently supported theoretically [1, 3, 2]. The new relation represents an important step beyond phenomenology, as

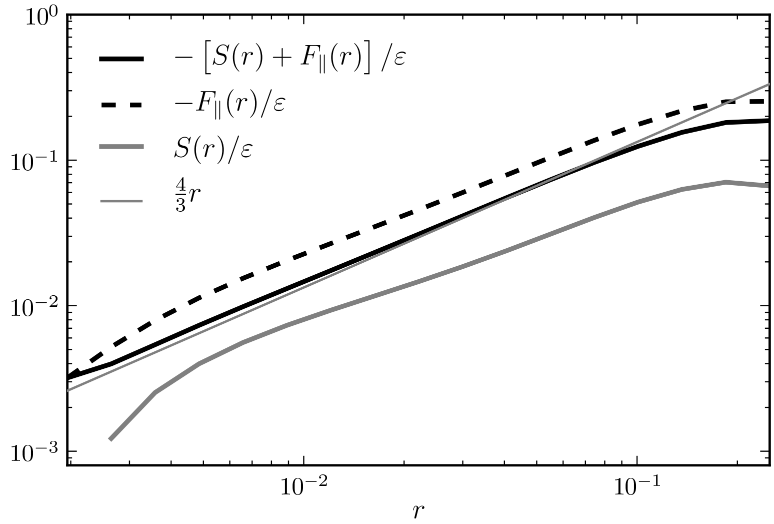

Figure 1. Scaling relation (1) for highly compressible turbulence at Mach 6 based on the simulation data [13]. For each of the 16 values of $r/\Delta \in [8, 160]$, where Δ is the linear grid cell size ($L = 1024\Delta$), approximately 2×10^9 random point pairs per flow snapshot were used to evaluate $S(r)$ and $F_{\|}(r)$.

it sheds light on the problem of universality in compressible turbulence and provides a way to quantitatively predict the energy injection rate based on observables. This result has important implications for interstellar turbulence, as constant energy transfer rates are observed in the interstellar medium over more than four decades in length scale [8].

The fourth-order scaling relation (1) is traditionally formulated in terms of the energy flux, but it is not the only compressible analog of Kolmogorov's four-fifth law. Another approximate fifth-order relation formulated in terms of fluxes and densities of conserved quantities [21] also reduces to the four-fiths law under the assumption of incompressibility [5]. Thus, at least two compressible analogs of the four-fifths law exist, consistent with the extension of the turbulent energy cascade picture to supersonic regimes. Meanwhile, only the fourth-order relation (1) is universal in a sense that its r.h.s. remains linear in the inertial range at all Mach numbers, including the weakly compressible, nearly incompressible, and incompressible regimes, see [21, 15] for more detail.

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References

[1] H. Aluie. *Phys. Rev. Lett.*, **106**(17):174502, 2011.
[2] H. Aluie. *Physica D*, 2013, in press; doi:10.1016/j.physd.2012.12.009.
[3] H. Aluie, S. Li, and H. Li. *Astrophys. J., Lett.*, **751**:L29, 2012.
[4] S. Banerjee and S. Galtier. *Phys. Rev. E*, 2013. submitted (arXiv:1301.2470).
[5] G. Falkovich, I. Fouxon, and Y. Oz. *J. Fluid Mech.*, **644**:465–472, 2010.
[6] S. Galtier and S. Banerjee. *Phys. Rev. Lett.*, **107**(13):134501, 2011.
[7] P. Goldreich *KITP Blackboard Lunch Talk,* http://online.kitp.ucsb.edu/bblunch/goldreich/, 2011.
[8] P. Hennebelle and E. Falgarone. *Astron. Astrophys. Reviews*, **20:55**:1–58, 2012.
[9] A. Hobbs, S. Nayakshin, C. Power, and A. King. *MNRaS*, **413**:2633–2650, 2011.
[10] A. Ingenito and C. Bruno. *AIAA Journal*, **48**:515–525, 2010.
[11] A. N. Kolmogorov. *Dokl. Akad. Nauk SSSR*, **30**:299, 1941.
[12] G. Kowal and A. Lazarian. *Astrophys. J., Lett.*, **666**:L69–L72, 2007.
[13] A. G. Kritsuk, M. L. Norman, P. Padoan, and R. Wagner. *Astrophys. J.*, **665**:416–431, 2007.
[14] A. G. Kritsuk, P. Padoan, R. Wagner, and M. L. Norman. *AIP Conf. Ser.*, **932**, 393–399, 2007.
[15] A. G. Kritsuk, R. Wagner, and M. L. Norman. *J. Fluid Mech.*, in preparation, 2013.
[16] A. F. J. Moffat and C. Robert. *Astrophys. J.*, **421**:310–313, 1994.
[17] D. E. Ogden, G. A. Glatzmaier, and K. H. Wohletz. *Earth and Planetary Sci. Lett.*, **268**:283–292, 2008.
[18] L. Pan, P. Padoan, and A. G. Kritsuk. *Phys. Rev. Lett.*, **102**(3):034501, 2009.
[19] W. Schmidt, C. Federrath, and R. Klessen. *Phys. Rev. Lett.*, **101**(19):194505, 2008.
[20] C. Schwarz, C. Beetz, J. Dreher, and R. Grauer. *Physics Letters A*, **374**:1039–1042, 2010.
[21] R. Wagner, G. Falkovich, A. G. Kritsuk, and M. L. Norman. *J. Fluid Mech.*, **713**:482–490, 2012.