Velocity level crossing statistics in wall bounded turbulent flows

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Abstract Statistics of dissipation and production at level-crossings of velocity fluctuations are analyzed through direct numerical simulations of a fully developed turbulent channel flow at Karman numbers of 180 and 360. Such statistics have not been determined correctly in past publications that reported biased results. It is shown that the spanwise level-crossings of wall normal velocity component dominate the dissipation, while, and despite the reflectional symmetry, the zero-crossings of the spanwise component contribute most to the production.

Consider the fluctuating streamwise velocity $u(x,y,z;t)$ where x,y,z are respectively the streamwise wall normal and spanwise directions. Consider the zero-crossings of u in the streamwise direction at a given y. The zero-crossing frequency of u is roughly (and exactly in case of a Gaussian signal) proportional to $\sqrt{\overline{(\partial u/\partial x)^2}}$ in wall-bounded flows (see [1] and [2] and the references within). That explains why it has been argued for a while that the zero-crossings of fluctuating velocity signals should largely contribute to the dissipation in, at least an isotropic turbulence. That the frequency is related to $\overline{(\partial u/\partial x)^2}$, does obviously not imply that the dissipation is locally important during the zero-crossings, and this point has not been entirely elucidated. The wall flow on the other hand can be considered locally isotropic only far away from the wall, say near the centerline of a channel, or the edge of a boundary layer. The aim of this investigation is to elucidate whether there is a direct or indirect relationships between the dissipation and the zero-crossings in wall bounded flows. We use a direct numerical simulations data basis obtained in a channel flow at a low Reynolds number (180, and 360 based on shear velocity). Consider the velocity component $u_I\left(x_i,t\right)$ and its zero-crossings in the x_J direction at a fixed time t. Using a suitably defined Dirac function at the zero-crossings points, we will first rigorously show that the ensemble-averaged dissipation $\bar{\varepsilon}_{zc\,I\,J}$ at the zero-crossing points of the component $u_I\left(x_i,t\right)$ in the direction x_J, is:

$$\bar{\varepsilon}_{zc\,I\,J}=\frac{E\left\{\varepsilon\left|u_{I,J}\right|\,\middle|\,u_I=0\right\}}{E\left\{\left|u_{I,J}\right|\,\middle|\,u_I=0\right\}} \tag{1}$$

where $\left|u_{I,J}(x_j)\right|$ is the absolute value of the derivative $\partial u_I/\partial x_J$ and $\varepsilon=\nu u_{i,j}\left(u_{i,j}+u_{j,i}\right)$ is the local dissipation, $u_{i,j}=\partial u_i/\partial x_j$ and the repeated indices imply summation (the components $i=1,2,3$ are also sometimes denoted by respectively u,v,w for the velocity components in x,y,z directions hereafter). Namely the mean dissipation at the zero-crossing points is the normalized mean of the dissipation times the absolute velocity derivative and not simply the ensemble averaged dissipation when the zero-crossings occur. The latter represents correctly the contribution to the dissipation at zero-crossing points, with:

$$\bar{\varepsilon}_{zc\,I\,J}=E\left\{\varepsilon\,\middle|\,u_I=0\right\} \tag{2}$$

only when ε and $u_{I,J}$ are independent, which obviously not the case here. The contribution of the zero crossings to dissipation (c_ε) can then easily be obtained from Eq. (1) while the *pseudo* contribution $c_{P\varepsilon}$ is related to (2). Relationships relating (1) and (2) will be provided in the paper. We determined c_ε and $c_{P\varepsilon}$ for three components of fluctuating velocity in the spanwise and longitudinal (homogeneous) directions at a given distance from the wall. Contrarily to what has been suggested before, the zero crossings of the *wall normal velocity* v in the *spanwise* direction z *contribute mostly to the dissipation* (figure 1) instead of longitudinal x crossings of the streamwise velocity u as shown in figure 2 (or any other combination, not shown here). The contribution to the dissipation near the points where the wall normal velocity changes sign in z is larger than 40% at $y^+\geq 20$, reaching 80% near the wall, as it is clearly seen in figure 2. ($^+$ denotes quantities scaled by the viscosity ν and shear velocity and y is the wall normal distance). In the logarithmic region all contributions decrease to about 30%. The small-scale structure of the turbulence close to the wall up to the buffer layer is locally dominated by v and z. That is the consequence of near wall coherent structures as we will show in the paper.

We will generalize these concepts in the second part of the paper by determining the statistics of a quantity q (such as the production, enstrophy…). One of the aims here is to investigate the level crossing characteristics in homogeneous planes y, and at a fixed time t, in the streamwise x and spanwise z directions. There are several ways to sample the data in this case. One can for example sample the velocity component $u_i(x,z;y,t)$ in the homogeneous x

direction along the constant z lines. In this case $u_i(x;z,y,t)$ is still a one-dimensional signal, and the relations obtained before apply directly by taking $u' = \partial u_i/\partial x$. One can also fix the sampling direction along x, and obtain the statistics of a quantity q at $u_i(z;x,y,t) = \ell$ level crossings by assigning $u' = \partial u_i/\partial z$. These characteristics are labeled as *directional*-level crossing properties hereafter.

These concepts have to be completed by contour crossings in the $x-z$ plane in order to establish a unique definition of the related statistics. The level crossings are then replaced by surface elevation of the velocity component $u_i(x,z;y,t)$ and the level crossings are then the level crossing contours $c(\ell) = \{(x,z) : u_i(x,z;y,t) = \ell\}$. The easiest way to determine the contour statistics of a quantity q is to introduce the generalized version of the Dirac function approach given in the equation (1). We will show that the average $\overline{q}_\ell$ of a given quantity q at level-crossing contours reads for:

$$\overline{q}_\ell = \frac{E\left\{q \times \sqrt{\left(\frac{\partial u_i}{\partial x}\right)^2 + \left(\frac{\partial u_i}{\partial z}\right)^2} \;|\; u_i = \ell\right\}}{E\left\{\sqrt{\left(\frac{\partial u_i}{\partial x}\right)^2 + \left(\frac{\partial u_i}{\partial z}\right)^2} \;|\; u_i = \ell\right\}} \qquad (3)$$

The u, v and w contour crossings statistics of local *production* $q = -uv\left(\partial \overline{U}/\partial y\right)$ will be subsequently discussed in the paper. One of the major observation, and although the spanwise velocity does not appear directly in q, is the large contributions to the Reynolds shear stress of w zero crossings in the low buffer layer. The production at u and v crossings is globally well predicted by a jointly Gaussian model beyond the buffer layer, but there are strong departures from normality in the viscous and low buffer regions. The contour statistics of the <u>discriminant and invariants</u> *of velocity gradient tensor* have attracting features and will also be presented.

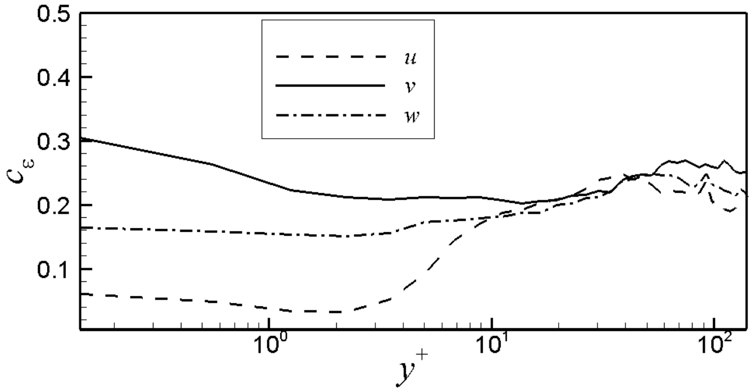

Figure 1. Contribution of *streamwise* zero crossings of the fluctuating velocity field to the total dissipation. Results inferred from direct numerical simulations at $Re_\tau = 180$.

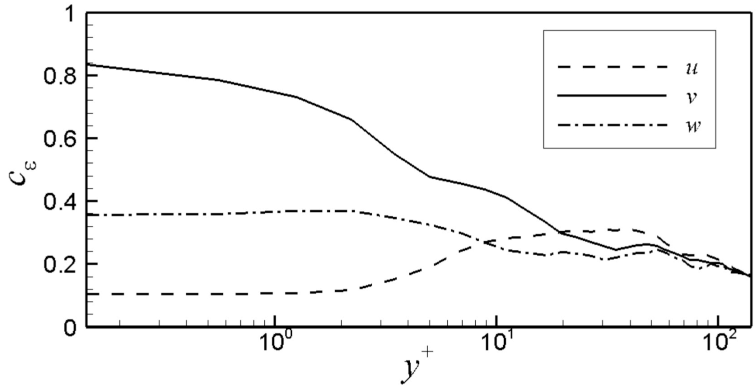

Figure 1. Contribution of *spanwise* zero crossings of the fluctuating velocity field to the total dissipation

References

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[2] Tardu S, 2012. A note on spatial zero-crossings of fluctuating velocity fields in wall bounded turbulent flows; *Springer Proceedings in Physics*, Volume 141, Part 1, 45-48.