EXPERIMENTAL INVESTIGATION ON 3D LAGRANGIAN COHERENT STRUCTURES IN THE LEFT VENTRICLE

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Abstract The complex Left Ventricle (LV) flow dynamics is known to be deeply affected by the vortical structures originated during diastolic inflow, as well as by their interaction (see [1] for a review). Here we present the results obtained in a laboratory model of the LV: Lagrangian Coherent Structures are identified and tracked thanks to the computation of Finite Time Lyapunov Exponents [2, 3] performed on the measured 3D experimental fields. The analysis on the formation and development of these structures provides important information on the LV flow.

EXPERIMENTAL APPARATUS

Investigations were performed in a laboratory model shown in Fig. 1a. The left ventricle is simulated, at the geometrical scale is 1:1, by means of a transparent sack made of silicone rubber, which is housed in a rectangular tank (A) provided with transparent walls for the optical access. The ventricle sack is mounted on a circular plate, 56 mm in diameter, having one inlet and one outlet (simulating aortic and mitral orifice, respectively) that are connected to a constant head reservoir and provided with two one-way valves. The ventricular volume change, $\Delta V(t)$, is assured by the motion of a piston, driven by a linear motor (C), while its time derivative, $Q(t)$, represents the flow rate through the mitral orifice, during the diastole, and through the aortic one, during the systole. The dynamic flow similarity between the real case and the model is assured by the matching of Reynolds and Womersley numbers:

$$Re = \frac{UD}{\nu} \qquad\qquad Wo = \sqrt{\frac{D^2}{T\nu}}$$

where D is the maximum diameter of the ventricle, U the peak velocity through the mitral orifice, T the period of the cardiac cycle, ν the kinematic viscosity of the working fluid. In order to reproduce physiological conditions Adopted parameters (listed in Table 1) are chosen. This experimental set-up correctly simulates the observed LV dynamics, as confirmed by previous two-dimensional measures performed on the same laboratory model [4, 5, 6].

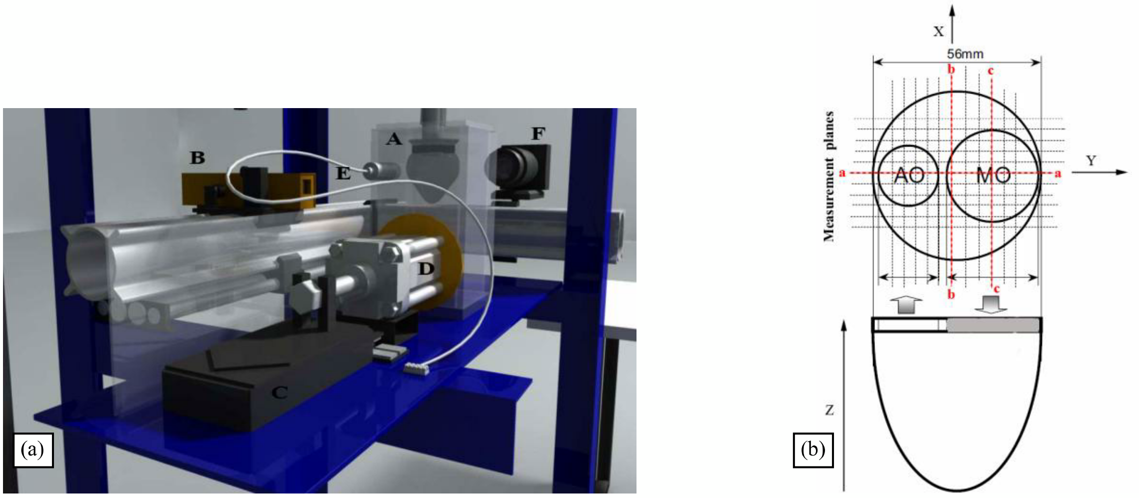

Figure 1. a) Sketch of the experimental setup; b) measurement planes.

Stroke Volume [ml]	T [s]	U [m/s]	Re [-]	Wo [-]
64	6	0.145	8322	22

Table 1. Experimental parameters.

The 3D ventricle flow is reconstructed using two sets of planar measurements acquired from two orthogonal views. Specifically, for each view, 50 cycles of flow images on 11 parallel planes 4 mm away are recorded (Fig. 1b). During each acquisition, the measurement plane is illuminated by a 12 W, infrared laser, while the working fluid inside the ventricle is seeded with neutrally buoyant particles, of about 30 µm in diameter. A high-speed digital camera (250 Hz, 1280×1024 pixel) is triggered by the motor to capture the time evolution of the phenomenon at known instants of the cycle. Images are then analyzed using a Feature Tracking algorithm [4], and velocity vectors are subsequently interpolated over a regular grid to reconstruct the 3D fields. Hence, the mean cycle is obtained by phases averages on the 50 experiments.

FINITE-TIME LYAPUNOV EXPONENTS AND LAGRANGIAN COHERENT STRUCTURES

Finite Time Lyapunov Exponents (FTLE) are a measure of the maximum linearized rate of growth of the distance among initially adjacent particles advected by the flow over a finite time interval. Trajectories can be integrated forward or backward in time: when a positive T* is considered, the FTLE measure separation forward in time, thus identifying repelling structures, on the contrary, if a negative T* is considered, FTLE measure separation backward in time, thus highlighting attracting structures. FTLE computation was performed on 3D velocity fields of the mean heart cycle by means of a public domain code, NEWMAN [7], adopting an integration time, T*, equal to the advection time scale, i.e. the ratio between the end-diastolic ventricle diameter and the peak mitral velocity, U.

Fig. 2a shows, as an example, the two dimensional section of the 3D backward FTLE corresponding to the end of the first diastolic wave . The FTLE ridges correspond to the attracting Lagrangian structure: the plot shows the upstream front of the jet originating from the mitral valve during the diastolic wave (A in the plot) and the signature of the trailing jet still connected to the mitral orifice (B). Since velocity fields do not cover the whole ventricle, data are missing in the right side of the plot. Despite this fact, one can observe the rolling of the right lobe of the vortex ring impinging on the ventricle wall (C). The corresponding 3D LCS is visualized in Fig. 2b using a FTLE isosurface. Although important information can be inferred from 2D FTLE [6], it is apparent how the comprehension of the complex asymmetric vortical structure highly benefits from the availability of 3D velocity fields. From the conjunct analysis of backward and forward FTLE fields it is possible to delimit the whole LCS, which emerge from the superposition of backward and forward FTLE ridges. With respect to traditional instant analyses techniques (vorticity, Q-criterion, etc.) the adopted method has the advantage to reveal the vortical structures boundaries and to encode fundamental Lagrangian description into each single field, conveying key information about the underlying mechanism of fluid transport. LCS evolution during the heart cycle will be presented, and further analysis will be performed in order to clarify their role in physiological LV flow as transport barriers and responsible for efficient mixing and recirculation.

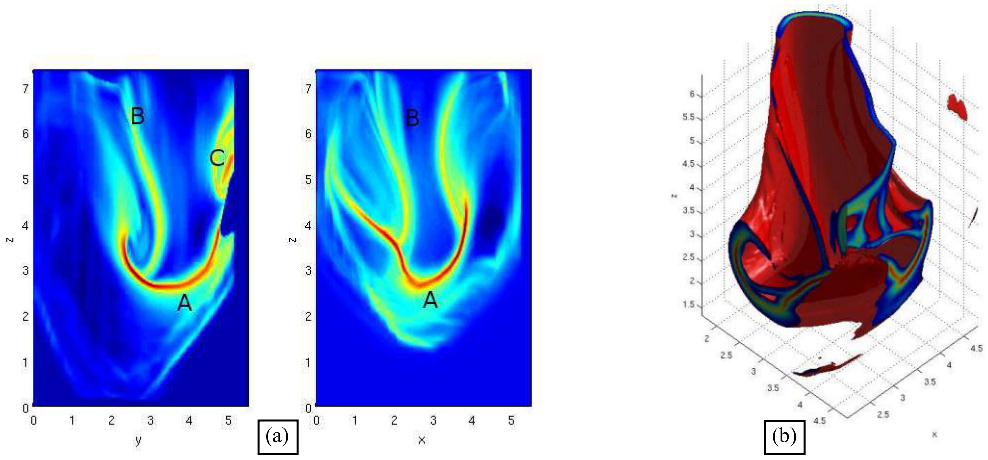

Figure 2. a) Color map of 2D sections of backward 3D FTLE computed at the end of the diastolic wave (sec. a-a and c-c in Fig. 1B for the left and right panel respectively). b) 3D representation of the FTLE field plotted in fig 2a.

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