HIGHER HARMONIC RESONANCE IN LATERALLY HEATED FLOW (LHF) WITH POISEUILLE FLOW COMPONENT (PFC)

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Abstract In this work we investigate pattern formation in the hierarchical transition to turbulence through the resonant nonlinear interaction between stationary states or travelling wave modes with wave numbers in the ratio 1:2, 1:3 and 1:4 for the case of LHF without PFC, while we also present results for 1:2 and 1:3 strongly nonlinear resonance in LHF with PFC.

INTRODUCTION

We consider fluid flow in a vertical layer with differentially heated side walls (Figure 1). In the presence of small temperature difference between the vertical boundaries, flow is parallel to the wall. Fluid near hotter and colder walls move upward and downward, respectively, because of the buoyancy effect. In this case heat transfers across the layer by conduction only, while for larger temperature differences (than a critical value of the Grashof number Gr_c, see below), convection by (strongly nonlinear) secondary and higher order flow enhances heat transfer across the layer.

If a pressure gradient in vertical direction exists, i.e. a Poiseuille component in the streamwise direction is present, then a traveling wave modulates efficiency of insulation of Ventilated Double Grazing (VDG), which is a primary target for European ecology, while simultaneously allowing for innovative engineering solutions.

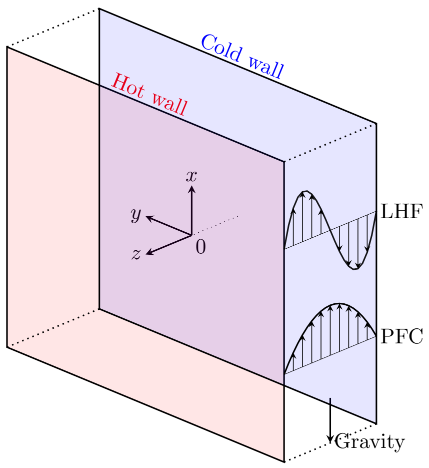

Figure 1. Geometry for VDG.

NONLINEAR SOLUTION AND THE RESONANCE IN PURE LHF

In this paper, we limit ourselves mainly to the study of two dimensional incompressible Boussinesq fluid flow for the cases Prandtl number Pr $(= 0, 0.025)$ with small Reynolds number $(0 \leq Re \leq 0.1)$. Herein we use the half gap width between two parallel plates, the maximum velocity of PFC across the midplane and the temperature difference between the walls as the definitions of nondimensional characteristic length, Re and Gr, respectively.

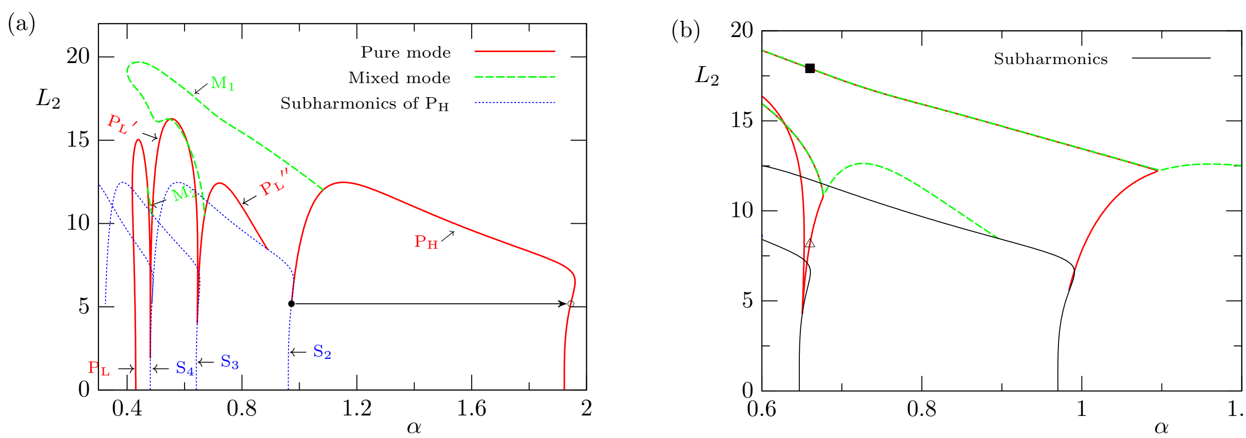

Figure 2. Bifurcation diagrams for $Gr = 1100$. (a) Pure LHF: $Pr = 0$, (b) VDG: $Pr = 0.025$, $Re = 0.1$.

The basic flow parallel to the walls is given by

$$\boldsymbol{U}_0 = (U_0(z), 0, 0) = \left(\frac{Gr}{6}\left(z^3 - z\right) + Re\left(1 - z^2\right), 0, 0 \right) \tag{1}$$

and just has a component in the vertical (streamwise) direction x (in Figure 1). Up to a critical Grashof number $Gr < Gr_c$ the heat mainly transfers between the walls by conduction, since the flow (1) is stable. At $Gr = Gr_c$ a two dimensional solution with a wave number α_c ($= 1.345$ for $Pr = 0$) in the $x - z$ plane (in Figure 1) appears[1, 2], as a result of the instability of the flow (1). After that two dimensional strongly nonlinear solutions can exist in a regime of α for $Gr > Gr_c$, so heat transfer across the vertical fluid layer is enhanced, since the convective flow carries much more heat than conduction.

In order to understand the characteristics of the two dimensional flow we depict a bifurcation diagram of a strength of the flow field L_2 [3] versus wave number α for $(Pr, Gr) = (0, 1100)$ in Figure 2 (a). There are six branches of solutions in either red (P_L, $P_L{}'$, $P_L{}''$ and P_H) or green (M_1 and M_2) and the subharmonic branches (S_2, S_3 and S_4), where each has a half, a third or a quarter of the wave number of P_H.

P_H bifurcates from the flow of equation (1) at $\alpha = 1.92$ and connects to S_2 at $\alpha = 0.973$ ($= \alpha_2$). This means that P_H ($\bullet$) becomes the solution with $2\alpha_2$ ($\circ$) in its branch at α_2 as a result of 1:2 spatial resonance. On the other hand (left) solution P_L bifurcated from $\alpha = 0.430$ has 1:4, 1:3 and 1:2 resonance with S_4, S_3 and S_2, respectively, in sequence (Figure 2(a)). The connections were also checked by the linear stability analysis performed on P_H.

$P_L{}''$ and P_H are connected with mixed mode solution M_1 (shown by dashed curve in green in Figure 2(a)) at $\alpha = 0.666$ and 1.08, respectively, while M_2 bridges solutions P_L and $P_L{}'$. Pure mode solutions have a certain symmetry in the flow field and are stationary. On the other hand, mixed mode solutions lose this symmetry and have phase velocity, in spite of no cross flow.

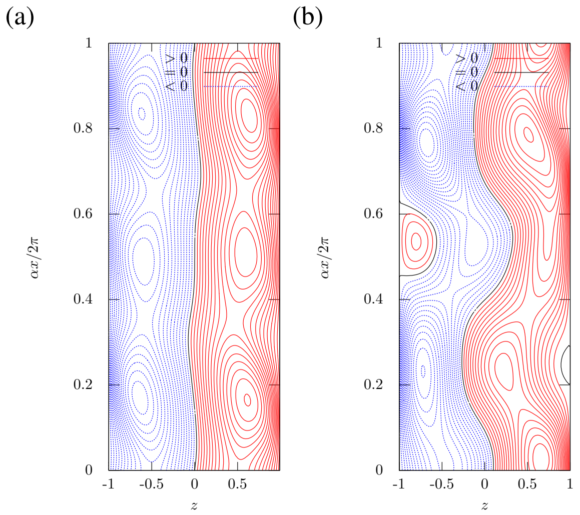

Figure 3. Contour levels of streamwise component of velocity (u) for $(Pr, Gr, Re) = (0.025, 1100, 0.1)$ (VDG) for solutions on (a) lower branch and (b) upper one correspond to solutions in red marked with $\triangle$ and $\blacksquare$ at $\alpha = 0.66$ in Figure2(b), respectively. Each level are drawn in red, black and blue for positive, zero and negative values, respectively.

NONLINEAR SOLUTION AND THE RESONANCE IN VDG

In the case of VDG, it can be found there is structural difference from LHF in bifurcation diagrams. As an example of VDG a bifurcation diagram for $(Pr, Gr, Re) = (0.025, 1100, 0.1)$ is depicted in Figure 2(b). Branch P_H in Figure 2(a) is split into two solutions in red and green, and there are two solutions corresponding to M_1. All solutions have phase velocity, even if solutions correspond to pure mode of LHF. It is confirmed in Figures 2(a) and (b) that there are multiple solutions in some regimes, and full bifurcation diagram is more complicated. In Figure 3 profiles of u, which is the streamwise component (x) of the velocity, in the $x - z$ plane are plotted to compare the solutions of $(Pr, Gr, Re, \alpha) = (0.025, 1100, 0.1, 0.66)$.

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